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Integrated Estimation/Guidance Design Approach for Improved Homing Against Randomly Maneuvering Targets

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Interceptor missiles, designed against aircraft, have substantial speed and maneuverability advantage over their targets. Thus, by exploiting the technological progress, even simple guidance concepts yielded satisfactory performance. For the interception of antisurface missiles, higher guidance precision is required. Using conventional guidance and estimation concepts, existing missile defense systems have demonstrated hit-to-kill accuracy against nonmaneuvering targets. Guaranteeing a similar performance against maneuvering targets can be achieved only if the estimation errors against such targets are minimized. This paper introduces a new, logic-based estimation/guidance algorithm, that explicitly uses the time-to-go in the estimation process and modifies the guidance law to reduce the consequence of estimation errors. The successful outcome of the new approach is illustrated by an extensive Monte Carlo simulation study.

I. Introduction

HISTORICALLY, guided interceptor missiles were designed against nonmaneuvering or moderately maneuvering aircraft-type targets. In such scenarios, the speed and the maneuverability of the missile largely exceed those of the target. Moreover, miss distances on the order of a few meters, compatible with the lethal radius of the missile warhead, were considered admissible due to the vulnerability of aircraft structures. New warfare concepts in antiballistic missile defense and ship defense scenarios involve the interception of antisurface missiles that attack high value targets. This task has presented an extreme challenge to the guided missile community. Tactical ballistic missiles, as well as modern antiship missiles, fly at very high speeds and their maneuvering potential in the atmosphere is comparable to that of the interceptors. Moreover, this potential can be made useful by a modest technical effort. Successful interception of an antisurface missile, carrying probably an unconventional warhead, requires a very small miss distance or even a direct hit (to hit a bullet with a bullet). Such hit-to-kill accuracy against targets emulating tactical ballistic missiles that fly on straight or ballistic trajectories has recently been demonstrated [1–3]. However, recent studies [4–6] have indicated that currently used guidance and estimation methods are unable to guarantee a satisfactory guidance accuracy against highly maneuvering targets that are expected in the future. To understand the origin of this deficiency, it is necessary to review the 50-year history of guided missiles design practice.

Guidance theory points out that the main error sources responsible for nonzero miss distances are: 1) noisy measurements, 2) nonideal dynamics of the guidance system, 3) the contribution of target maneuvers, 4) limited missile maneuverability, leading to saturation. Nevertheless, simulation studies and flight tests have demonstrated that adequate maneuverability advantage of the interceptor can make the resulting miss distances sufficiently small.

All known missile guidance laws used at present were developed based on a linearized kinematical model and a linear quadratic optimal control concept (with unbounded control), so that the limited maneuver potential of the interceptor has not been explicitly taken into account. Advanced guidance laws have included the effects of nonideal dynamics of the guidance system and the contribution of target maneuvers in the generalized zero-effort miss distance and used a time-varying gain schedule [7]. To evaluate the contribution of the target maneuvers, their current value and future evolution must be known. Because the current target maneuver cannot be directly measured, it has to be estimated. In most cases, a constant target maneuver has been assumed in the estimator's structure. Theoretically, if the assumption on the target behavior is correct, the measurements are ideal and the lateral acceleration of the interceptor does not saturate, such a guidance law can reduce the miss distance to zero. In practice, if the interceptor/target maneuver ratio is sufficiently high, the inevitable saturation occurs only very close to the end of the interception and the resulting miss distance becomes negligible.

In realistic interception scenarios with noise-corrupted measurements, an estimator has become an indispensable element of the guidance system and the homing performance of the interceptor missile has been limited by the estimation accuracy. Although, for realistic interceptor guidance scenarios with noise-corrupted measurements, bounded controls, and saturated state variables, as well as non-Gaussian random disturbances, the validity of the separation theorem [8] (stating that the estimation and control processes can be separately optimized) has never been proved, it has been of common practice to design the estimators and missile guidance laws independently. The estimators were simple Wiener or Kalman filters and the guidance laws were derived using simplified (linearized and planar) deterministic models. In most cases, such convenient design approach had been acceptable, because it succeeded in satisfying the performance requirements, due to the substantial maneuverability advantage of guided missiles over their manned aircraft targets. Applying this suboptimal approach also to the interception of antisurface missiles with high maneuverability, results in unsatisfactory homing performance [9].

In cases in which the separation theorem does not hold, a generalized separation property was asserted [10], stating that the

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estimator can be designed independently of the controller, but the derivation of the optimal control function has to be based on the conditional probability density function (conditioned on the measurement history) of the estimated state variables. Unfortunately, this very important idea had not been followed by a rigorous practical implementation approach for a very long period, and had not been applied in any known control design including guided missiles. Nevertheless, several attempts outlining new implementation ideas have been proposed in recent years. The first one in this direction was the development of a guidance law partially compensating the estimation delay (denoted as DGL/C) [9]. This, however, was only an approximation that neglected the stochastic features of the problem, associated with the noisy measurements, and reemphasized the need for improved estimation performance. Further advances in this direction have been recently introduced in [11,12], that address a linear problem with Gaussian noises. In these works the separation theorem is not applicable because an acceleration saturation is assumed. In [11], the nonlinear saturation element is replaced by a linear representation through which the conditional probability distribution function affects the guidance. In [12] the stochastic Hamilton–Jacobi–Bellman equation (HJB) is solved directly, including the nonlinear saturation effect. The idea of using an adaptive scheme of estimation and guidance, using a modified GLR detector, has been recently proposed in [13,14], showing some improvement of the homing performance. A first comprehensive attempt at a new, unified approach to integrating guidance with estimation in a generalized nonlinear non-Gaussian framework has been recently presented in [15]. Based on the guidelines of the generalized separation theorem [10], and extending the conventional notion of the reachability set [16–18], the proposed approach integrates the conditional probability density function into the guidance algorithm. However, the approach of [15] is computationally intensive, and still needs to be thoroughly verified through extensive experimentation. It should be noted that none of the above methods could be applied to the problem of intercepting highly maneuverable antisurface missiles in a satisfactory manner.

The objective of this paper is to report the results of a recently completed multiyear investigation that outlines a new integrated logic-based estimation algorithm that achieves substantial homing improvement. The remainder of this paper is organized as follows. In the next section, the interception problem of maneuvering antisurface missiles is formulated. This is followed by a brief review of a deterministic optimal interceptor guidance concept and its implementation in a scenario of noise-corrupted measurements. In Sec. IV the difficulties in finding a feasible optimal estimator for this task are discussed. The new idea of an integrated logic-based estimation/guidance algorithm, alleviating the difficulties, is introduced in Sec. V, which includes results of a large set of Monte Carlo simulations. Concluding remarks are presented in the last section.

II. Problem Statement

A. Scenario Description

For the sake of research efficiency (simplicity, repeatability, and reduced computational load), the analysis reported in this paper is performed using a planar (horizontal) constant speed model. Such a model can represent approximately the interception of a low flying cruise missile. Validation of the approach in a generic three-dimensional endoatmospheric ballistic missile defense (BMD) scenario with time-varying parameters (velocities and acceleration limits), requiring a large set of additional simulations, will be presented in a follow-up paper.

It is assumed that the homing endgame scenario starts shortly before interception, as soon as the onboard seeker of the interceptor succeeds in locking on the target. The relative geometry is close to a head-on engagement. It is assumed that at this moment the initial heading error, with respect to a collision course, is small and neither the interceptor nor the target is maneuvering. These assumptions permit a linearization of the interception geometry.

B. Information Structure

A basic assumption underlying the new concept is that the time-to-go, which constitutes a critical piece of information, is available to the homing interceptor. The time-to-go can either be computed by an interceptor equipped with an active seeker that can measure range and range-rate with good accuracy, or, in the case of a passive seeker, it has to be provided by the launching platform. Measurements of the line of sight angle are also available, but these are corrupted by a zero-mean, white Gaussian angular noise. The interceptor's own acceleration is accurately measured, but the target acceleration has to be estimated based on the available measurements. The target has no information on the interceptor, but, being aware of an interception attempt, it can start applying evasive maneuvers at any time, randomly changing the direction of the maneuver.

C. Lethality Model

The objective of the interception is the destruction of the target (the attacking antisurface missile). In the reported investigation, the probability of destroying the target is determined by the following simplified lethality function

$$P_d(M, R_k) = \begin{cases} 1 & M \le R_k \\ 0 & M > R_k \end{cases} \tag{1}$$

where R_k is the lethal (kill) radius of the warhead and M is the miss distance. This model assumes an overall reliability of 100% of the entire guidance system.

D. Performance Index

The natural (deterministic) performance index of the interception engagement is the miss distance. Because of the noisy measurements and the random target maneuvers, the miss distance becomes a random variable with an a priori unknown probability distribution function. A large number of Monte Carlo simulations can provide an empirical estimate of the cumulative probability distribution function, that allows comparing the homing performances of different guidance systems. Based on the lethality function of Eq. (1), the efficiency of a guided missile strongly depends on the lethal radius R_k of its warhead.

One figure of merit is the single shot kill probability for a given warhead, defined by

$$\text{SSKP} = E\{P_d(M, R_k)\} \tag{2}$$

where the mathematical expectation is computed with respect to the miss distance random variable, which is a function of the measurement noise and the random target maneuver. The objective of the guidance system is to maximize this value.

An alternative figure of merit is the smallest possible lethal radius $R_k(\kappa)$ that guarantees a predetermined probability of success κ [i.e., $\text{SSKP}[R_k(\kappa)] = \kappa$]. In several recent studies [19–21] the required probability of success has been set to $\kappa = 0.95$, yielding the following performance index

$$J = R_k(0.95) \tag{3}$$

to be minimized by the guidance system.

E. Equations of Motion

The analysis of an interception endgame is based on the following set of simplifying assumptions:

1) The engagement between the interceptor (pursuer) and the maneuvering target (evader) takes place in a horizontal plane.

2) Both the interceptor and the maneuvering target have constant speeds V_j and bounded lateral accelerations, $|a_j| \le (a_j)^{\max}$, for $j = E, P$.

3) The maneuvering dynamics of both vehicles can be approximated by first-order transfer functions with time constants τ_P and τ_E, respectively.

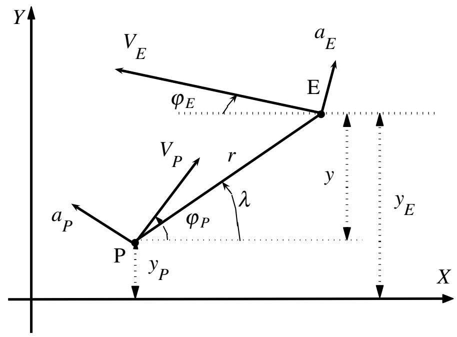

Fig. 1 Interception geometry.

4) The relative interception trajectory can be linearized with respect to the initial line of sight, with which the X-axis of the coordinate system is aligned.

In Fig. 1 a schematic view of the endgame geometry is shown. Note that the respective velocity vectors are generally not aligned with the reference line of sight. The angles φ_P and φ_E are, however, small. Thus, the approximations $\cos(\varphi_i) \approx 1$ and $\sin(\varphi_i) \approx \varphi_i$ $(i = P, E)$, are uniformly valid and coherent with assumption 4). Based on assumptions 2) and 4), the final time of the interception can be computed for any given initial range R_0 of the endgame by

$$t_f = R_0/V_c \tag{4}$$

where V_c is the closing speed. Since the angles φ_P and φ_E are assumed to be small, then, in a head-on engagement

$$t_f = R_0/(V_P + V_E) \tag{5}$$

The time-to-go is defined as

$$t_{\text{go}} = t_f - t \tag{6}$$

The state vector in the equations of relative motion normal to the reference line is

$$X = (x_1, x_2, x_3, x_4)^T = (y, \mathrm{d}y/\mathrm{d}t, a_E, a_P)^T \tag{7}$$

where

$$y(t) \triangleq y_E(t) - y_P(t) \tag{8}$$

The corresponding equations of motion and the respective initial conditions are

$$\dot{x}_1 = x_2, \qquad x_1(0) = 0 \tag{9a}$$

$$\dot{x}_2 = x_3 - x_4, \qquad x_2(0) = V_E\varphi_{E_0} - V_P\varphi_{P_0} \tag{9b}$$

$$\dot{x}_3 = (a_E^c - x_3)/\tau_E, \qquad x_3(0) = 0 \tag{9c}$$

$$\dot{x}_4 = (a_P^c - x_4)/\tau_P, \qquad x_4(0) = 0 \tag{9d}$$

where a_E^c and a_P^c are the commanded lateral accelerations of the target and the interceptor, respectively. The players' acceleration commands are conveniently modeled as

$$a_E^c = a_E^{\max} v, \qquad |v| \le 1 \tag{10a}$$

$$a_P^c = a_P^{\max} u, \qquad |u| \le 1 \tag{10b}$$

where v and u are the normalized acceleration commands of the target and the pursuer, respectively.

The nonzero initial conditions $V_E\varphi_{E_0}$ and $V_P\varphi_{P_0}$ represent the respective initial velocity components not aligned with the initial (reference) line of sight. By assumption 4) these components are small relative to the components along the line of sight. Equations (9) can be written in a compact form as a linear, time invariant, vector differential equation

$$\mathrm{d}X/\mathrm{d}t = AX + Bu + Cv \tag{11}$$

The problem involves two nondimensional parameters of physical significance: the pursuer/evader maximum maneuverability ratio

$$\mu \triangleq (a_P)^{\max}/(a_E)^{\max} \tag{12}$$

and the evader/pursuer time constant ratio

$$\varepsilon \triangleq \tau_E/\tau_P \tag{13}$$

The miss distance (the deterministic cost function of the interception), can be written as

$$M = |DX(t_f)| = |x_1(t_f)| \tag{14}$$

where

$$D = (1, 0, 0, 0) \tag{15}$$

F. Problem Formulation

There is a fundamental deficiency in formulating the interception of a maneuverable target as an optimal control problem. Target maneuvers are independently controlled. Since future target maneuver time history (or strategy) cannot be predicted, the optimal control formulation is not appropriate. The scenario of intercepting a maneuverable target has to be formulated as a zero-sum differential game of pursuit–evasion [22,23]. In such a formulation, there are two independent controllers and the cost function is simultaneously minimized by one of them and maximized by the other. Based on the above outlined assumptions and formulation, deterministic zero-sum pursuit–evasion game models can be solved. The game solution provides simultaneously the interceptor's guidance law (the optimal pursuer strategy), the worst target maneuver (the optimal evader strategy) and the resulting guaranteed miss distance (the saddle-point value of the game). An optimal guidance law based on the solution of a perfect information linear game with bounded control was published in the past [24] and is briefly reviewed in the sequel.

III. Game Optimal Guidance Law

The game model assumes planar geometry, constant velocities and fixed acceleration limits [24]. The set of assumptions 1)–4) allows casting the problem into the canonical form of linear games, from which a reduced-order game with only a single state variable, the zero effort miss distance (denoted Z), is obtained. As the independent variable of the problem, the time-to-go (t_{go}), defined by Eq. (6), is selected. The solution of this game is determined by the two parameters of physical significance and defined by Eqs. (12) and (13).

The guidance law based on the game optimal pursuer strategy, denoted as DGL/1, is of a bang–bang type

$$u^* = \operatorname{sign}\{Z\}, \quad \forall\ Z \neq 0 \tag{16}$$

u^* being the normalized optimal control of the pursuer (interceptor). The explicit expression for Z is

$$Z = x_1 + x_2 t_{\text{go}} - \Delta Z_P + \Delta Z_E \tag{17a}$$

where

$$\Delta Z_P = x_4(\tau_P)^2[\exp(-\vartheta_P) + \vartheta_P - 1] \tag{17b}$$

$$\Delta Z_E = x_3(\tau_E)^2[\exp(-\vartheta_E) + \vartheta_E - 1] \tag{17c}$$

are the own and target acceleration contributions to Z, respectively,

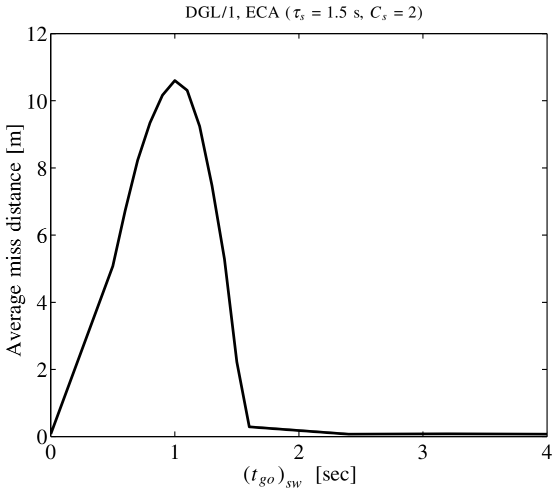

Fig. 2 Homing performance of DGL/1 against bang–bang target maneuvers.

and $\vartheta_P = t_{\text{go}}/\tau_P$ and $\vartheta_E = t_{\text{go}}/\tau_E$. The guaranteed miss distance depends on the parameters of the game (μ, ε) and can be made zero for all initial conditions of practical importance if both $\mu > 1$ and $\mu\varepsilon \geq 1$.

Implementation of DGL/1 requires perfect knowledge of the zero-effort miss distance, which includes also x_4, the current lateral acceleration of the target. Since this variable cannot be directly measured, it has to be estimated based on noise-corrupted measurements. This process involves an inherent delay. If the interceptor uses DGL/1, derived from the perfect information game solution [24], a smart target can take advantage of the estimation delay and achieve a large miss distance by adequate optimal maneuvering [25] as illustrated in Fig. 2, even if the game parameters are such that the guaranteed miss distance should be zero. Figure 2 represents the results of a large set of planar (horizontal) simulations against a target performing bang–bang type evasive maneuvers with a single, randomly timed acceleration direction reversal (switch) during the endgame. Such type of an evasion maneuver was found to be the optimal one for interception avoidance [26]. The figure shows the average miss distance of 100 Monte Carlo runs as a function of $(t_{\text{go}})_{\text{sw}}$, the timing of the switch in the target maneuver direction. The data used for these simulations are given in Table 1.

In the simulations a typical Kalman filter augmented with a shaping filter is used. Such a shaping filter, driven by a zero-mean white noise, represents random target maneuvers [27]. The shaping filter selected for this case is based on an exponentially correlated acceleration (ECA) model, suggested by Singer [28]. Such a shaping filter has first-order dynamics with two tuning parameters, the correlation time of the maneuver τ_s and the level of the assumed process noise, expressed by its standard deviation $\sigma_s = a_E^{\max}/C_s$. In this example the parameters of the shaping filter are $\tau_s = 1.5$ s and $C_s = 2$.

The main reason for the degraded homing performance is the inherent delay introduced in the estimation of the target maneuver by the convergence time of the estimator. DGL/1 can correct the error created by the delay only if the change of the acceleration command occurs in the early part of the endgame [$(t_{\text{go}})_{\text{sw}} > 1.6$ s in the present example]. In this case sufficient time remains until intercept, the estimated acceleration converges and the guidance law receives sufficiently accurate values of the zero-effort miss distance early enough for achieving good precision.

The value of the delay can be reduced by increasing the bandwidth of the estimator by selecting different tuning parameters of the shaping filter. Using such shaping filter, the large miss distances associated with command switches occurring near the end of the interception will be reduced, at the expense of less efficient filtering that will lead to larger residual converged estimation errors. This will give rise to increased miss distances for acceleration command changes occurring in the early parts of the endgame. For improved homing performance, both the estimation delay and the variance of the converged estimation error have to be reduced. To achieve this objective, extensive simulation studies have been aimed at searching for an improved estimation scheme, which is suitable for interception endgame problems.

Table 1 Horizontal endgame parameters

Parameter	Value
Interceptor velocity	$V_P = 2300$ m/s
Target velocity	$V_E = 2700$ m/s
Interceptor lateral acceleration limit	$a_P^{\max} = 20$ g
Target lateral acceleration limit	$a_E^{\max} = 10$ g
Time constant of the interceptor	$\tau_P = 0.2$ s
Time constant of the target	$\tau_E = 0.2$ s
Initial range	$R_0 = 20$ km
Endgame duration	$t_f = 4$ s
Measurement noise standard deviation	$\sigma_{\text{ang}} = 0.1$ mrad
Measurement rate	$f = 100$ Hz

IV. On Optimal Estimation

In the search for a suitable optimal estimator for the task of intercepting randomly maneuvering targets, several difficulties are encountered. The first one is of a conceptual nature. For linear systems with zero-mean, white Gaussian measurement and process noises, the Kalman filter [29], based on the correct model of the system dynamics, is the minimum variance optimal estimator. The measurement noise used in interception simulations has indeed such characteristics, but the representation of random target maneuvers as the output of a shaping filter driven by a zero-mean, white Gaussian noise, is only an approximation [27]. Moreover, each type of target maneuver requires a different shaping filter approximation.

Since target maneuver dynamics is not ideal, the target acceleration is regarded as a state variable, as part of the interception model. The disturbance inputs are the random acceleration commands, that can be discontinuous, representing a random jump process. They are bounded and certainly neither white nor Gaussian.

In some recent papers [30,31] it was shown that in such cases the optimal estimator is of infinite dimension. Thus, every computationally feasible (finite dimensional) estimator can be, at best, only a suboptimal approximation and the search for a feasible optimal estimator associated with interceptor guidance is not a well-posed problem. Similarly, it should be of no surprise that the certainty equivalence principle and the associated separation theorem [8], both involving the concept of optimality, have never been proven valid for the interception of randomly maneuvering targets. Not being able to rely on separate optimization of the estimator and the guidance law, one should search for other, efficient, feasible approaches.

The requirements to reduce both the estimation delay and the variance of the converged estimation error, mentioned at the end of the previous section, are contradictory. The delay associated with identifying a rapid target maneuver change is composed of the maneuver change detection time and the estimator's response time. Short detection time comes at the price of high false alarm rate. Short response time requires large bandwidth, which is associated with large estimation errors. For small estimation errors a narrow bandwidth is needed, which leads to slower response.

This controversy raises the question: can a single Kalman filter-type estimator satisfy the contradictory requirements of homing accuracy? Extensive Monte Carlo simulations have shown that no such estimator can be globally optimal for all guidance laws/interception scenarios, and there is no unique optimal Kalman filter-type estimator/guidance law combination that is suitable for all feasible target maneuvers [21]. Thus, a heuristic approach, based both on the insight generated by the extensive simulation results, as well as on control engineering intuition, is adopted herein. Some elements of the new approach have already been introduced in two conference papers [19,20].

V. New Approach

A. Integrated Estimation/Guidance Strategy

Because it was observed that no single Kalman filter-type estimator can satisfy the requirements of homing accuracy, the various tasks performed by a classical estimator have to be separated and assigned to different elements within a corporate estimation system. The main task, directly affecting the homing accuracy, is the estimation of the state variables (including the target acceleration) involved in the guidance law. This task can be performed satisfactorily by a narrow bandwidth filter, if and only if the correct model of the target maneuver is available. Thus, at the initial part of the endgame, the first task to be carried out is model identification, using, for example, a static multiple-model adaptive estimator (MMAE) [32]. Specifically, the MMAE (or any other technique used for this purpose) is used for discriminating between a piecewise-constant maneuver (e.g., bang–bang) and a time-varying maneuver (e.g., spiral). The filters for this task should be of a rather large bandwidth, in order to complete the model identification as fast as possible.

Because, in a planar scenario, a bang–bang-type maneuver is the most effective for evasion [24,26], the present paper focuses on it. Such a maneuver can be characterized as a piecewise-constant lateral acceleration maneuver, for which the model has to include its amplitude and current direction. If the anticipated direction reversal (switch) occurs sufficiently far from the end of the interception, there is sufficient time for the filter to converge after identifying the maneuver. The DGL/1 guidance law, using a sufficiently accurate value of the target acceleration, achieves small miss distances, as shown in Fig. 2. However, if the switch occurs near the end of the interception (e.g., at $t_{\mathrm{go}} = 1.0$ s), a very large miss distance is generated due to the estimation delay.

In an earlier paper [33], a multiple-model estimator, where each model assumes a different timing of the switch, is presented. Using a single estimator that is ideally tuned to the correct switch virtually eliminates the delay, as shown in Fig. 3. This improved estimation performance yields excellent homing performance, as can be seen in Fig. 4, which presents the cumulative probability distribution of the miss distance, obtained from 100 Monte Carlo runs for $(t_{\mathrm{go}})_{\mathrm{sw}} = 1.0$ s.

Moreover, even if the switch occurs shortly after the time anticipated by the estimator, similarly good homing performance is obtained, as illustrated in Fig. 5. This figure depicts the average miss distances of 100 Monte Carlo runs for three different values of $(t_{\mathrm{go}})_{\mathrm{sw}}$, as a function of $\Delta(t_{\mathrm{go}})_{\mathrm{sw}}$, the difference between the tuning time of the estimator and the true value of $(t_{\mathrm{go}})_{\mathrm{sw}}$:

$$\Delta(t_{\mathrm{go}})_{\mathrm{sw}} = (t_{\mathrm{go}})_{\mathrm{tune}} - (t_{\mathrm{go}})_{\mathrm{sw}} \tag{18}$$

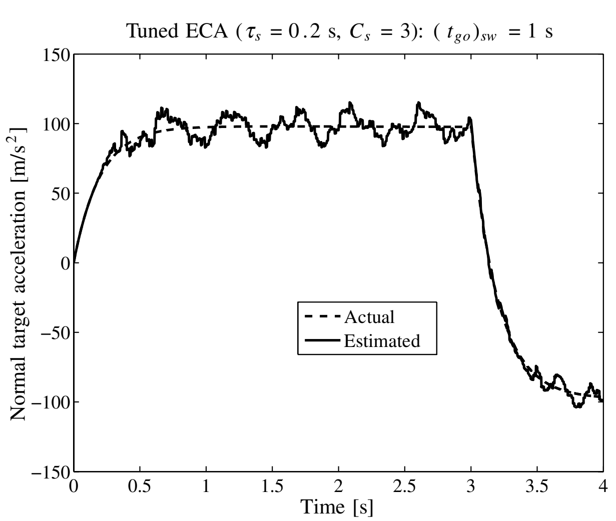

Fig. 3 Target acceleration estimation performance of an ideally tuned estimator.

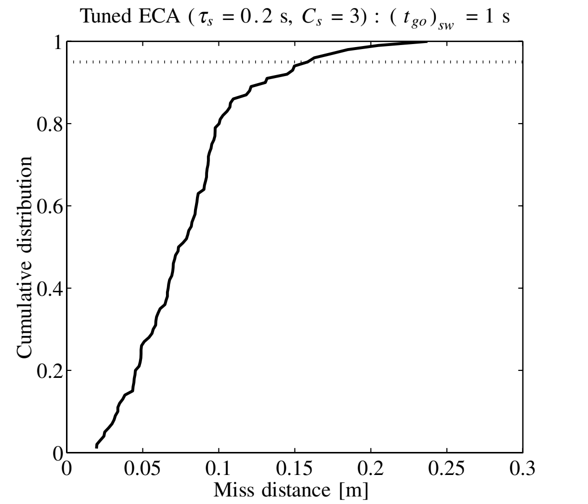

Fig. 4 Cumulative miss distance distribution of DGL/1 with an ideally tuned estimator.

Figure 5 shows small miss distances and a surprising robustness, allowing the use of only very few tuned estimators for covering the range of interest. The estimators for this evaluation employ, similar to the one used for generating Fig. 4, ECA shaping filters with a relatively large bandwidth ($\tau_s = 0.2$ s, $C_s = 3.0$).

If the event of the switch in the target acceleration command can be detected sufficiently fast, this robustness property suggests (for the example using the data of Table 1 and the results shown in Figs. 2–5), a logic-based estimation/guidance strategy as a function of time-to-go. This strategy consists of the following phases:

1) Until the identification of the target maneuver type, a narrow bandwidth estimator is used, along with a guidance law that does not use the target acceleration to compute the zero-effort miss distance. Termed DGL/0, this guidance law thus uses the following expression for the zero-effort miss distance [34]:

$$Z = x_1 + x_2 t_{\mathrm{go}} - \Delta Z_P \tag{19}$$

2) Once the direction of the constant maneuver has been identified, the guidance law is changed to DGL/1, preserving the same estimator.

3) If a jump in the direction of the target maneuver command is detected before a critical time-to-go ($t_{\mathrm{go}} = 1.6$ s in the present example), this estimator is maintained until the end. Depending mostly on the narrow-band estimator's performance and the pursuer

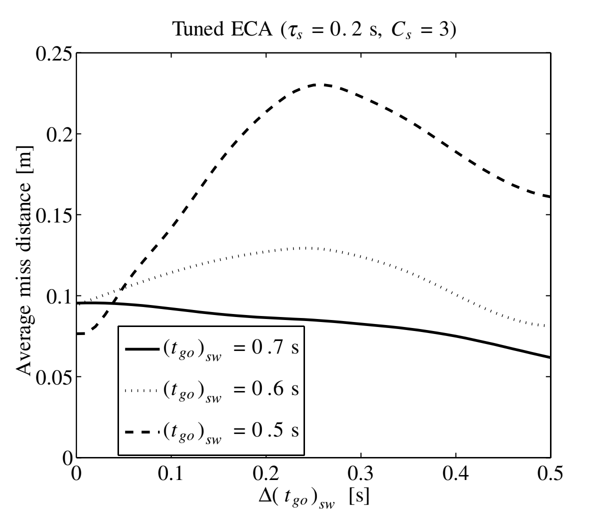

Fig. 5 Average miss distances with tuned estimators for bang–bang target maneuvers.

dynamics, the critical time-to-go is determined using offline simulations.

4) If the jump in the direction of the target maneuver command is detected after the critical time-to-go, the narrow bandwidth estimator that provides the input of the guidance law is replaced by a wide-bandwidth filter that is tuned to the nearest earlier switch time from a preselected set of switch times. After jump detection, the activated estimator remains unchanged. Because the estimation delay with a tuned estimator is negligible, the guidance law used with these estimators is also DGL/1. In the example presented in this paper, the preselected set of switch times consists of just three elements: $(t_{go})_{sw} = 1.6$, 1.0, and 0.5 s (hence, just three tuned filters are used). This set was found sufficient for covering the entire range of interest.

The new estimation/guidance strategy was tested in an extensive Monte Carlo simulation study. The study included 40 evenly spaced switch times over the entire 4-s duration of the benchmark endgame, using 100 noise samples for each switch time. The results, based on the assumption of ideal detection, are shown in Fig. 6, displaying the miss distance cumulative probability distribution. These results are very close to satisfying the hit-to-kill requirement.

Because an ideal detection of the jump in the direction of the target maneuver command is impossible, the Monte Carlo simulations were repeated assuming small detection delays of 0.05 and 0.1 s. The miss distance cumulative probability distributions for these two cases are also shown in Fig. 6. As can be observed from Fig. 6, a detection delay of 0.05 s has only a minor effect, whereas a delay of 0.1 s causes a more significant performance degradation, mainly for maneuver switches near the end of the interception. These results strongly emphasize the need for a fast jump detector, which has to be developed, as an additional element of the integrated estimation system. Figure 6 also includes the results presented in [20], already using the ideas of task separation and the explicit application of the time-to-go in the estimation process, but employing the DGL/C guidance law with tuned estimators assuming ideal jump detection. The comparison clearly indicates the improvement achieved by relying on DGL/1 in the entire endgame after model identification, even if the jump detection is not ideal.

To alleviate the negative effects of the detection delay, two important modifications are introduced in the DGL/1 guidance law at the last phase of the endgame. These modifications are presented next.

B. Guidance Law Modifications

It was observed that, due to the detection delay and the remaining short time, the interceptor is unable to reach its maximum lateral acceleration and reduce the guidance error generated during the delay. This deficiency can be corrected by increasing the lateral acceleration command for small values of time-to-go.

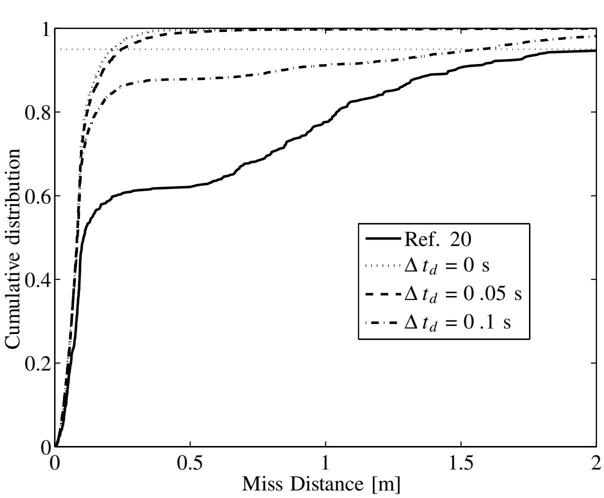

Fig. 6 Cumulative miss distance distributions with logic-based tuned estimators.

The increase in the commanded acceleration gain is expressed, for $t_{go} \le (t_{go})_{sw}$, by

$$a_P^c = a_P^c(t_{go}, k) = \frac{a_P^{\max} \operatorname{sign} Z}{1 - k \exp(-t_{go}/\tau_p)} \tag{20}$$

where the parameter k is selected to satisfy

$$|a_P(t_f, k)| = a_P^{\max} \tag{21}$$

From Eqs. (20) and (21) it is clear that $k < 1$; otherwise the gain would be infinite. The value of k depends on $(t_{go})_{sw}$ and the value of a_P at that very moment. The effect of this modification is illustrated in Fig. 7, comparing the acceleration time histories generated by the classical command of Eq. (16) (dashed line) and the command of Eq. (20) (solid line).

A further improvement is achieved by introducing a time-varying dead-zone version of the signum function in the DGL/1 guidance law for the period when the tuned estimators are used:

$$\operatorname{sign}_{DZ}(Z) = \begin{cases} 1 & Z > Z_{DZ} \\ 0 & |Z| \le Z_{DZ} \\ -1 & Z < -Z_{DZ} \end{cases} \tag{22}$$

where

$$Z_{DZ} \triangleq A_{DZ} \exp[-b_{DZ}(t_f - t_{go})] \tag{23}$$

In Eq. (23), the parameter A_{DZ} guarantees that the dead zone is sufficiently small, and b_{DZ} is the exponential decay rate of the dead zone. Both of these tuning parameters are determined using offline simulations for the set of expected worst-case scenarios. This modification reduces the error created during the period of detection delay, as illustrated in Fig. 8. The dead zone is used only in the interval $1.0\ \mathrm{s} > t_{go} > 0.2\ \mathrm{s}$ until the switch is detected. In the simulations, the values of $A_{DZ} = 50$ m and $b_{DZ} = 1\ \mathrm{s}^{-1}$ were selected.

By applying the two modifications expressed by Eqs. (20–23), a major improvement in the homing performance is achieved, as can be clearly seen in the cumulative distributions presented in Fig. 9.

Table 2 presents various figures of merit for the homing performance in the horizontal constant speed interception endgame: the average miss distance r_{av}, the maximum miss distance for SSKP $= 0.95$, r_{95}, and $p_{0.5}$, the kill probability for warhead lethality radius of 0.5 m. The results summarize the effect of the detection delay and the two new modifications of the guidance law. One can see that the modifications succeed in compensating for the performance degradation due to the imperfect detection.

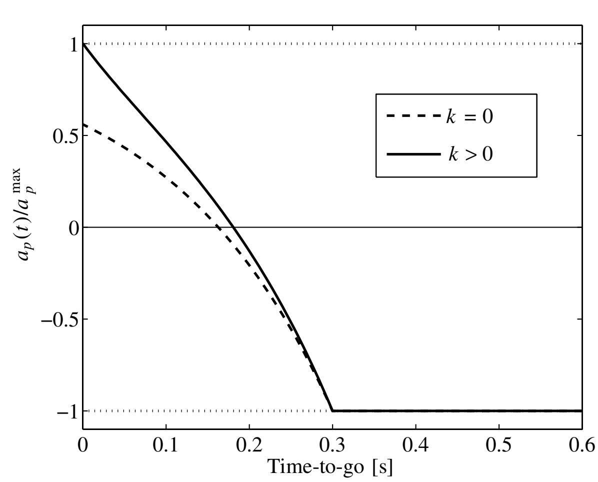

Fig. 7 The effect of command gain enhancement.

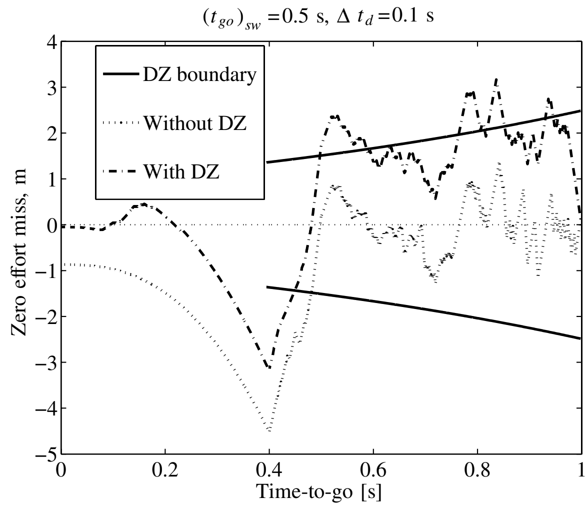

Fig. 8 The effect of time-varying dead zone.

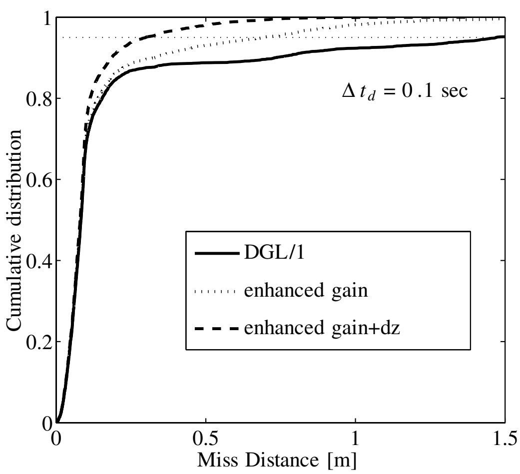

Fig. 9 Cumulative miss distance distributions with guidance law modifications.

VI. Conclusions

In this paper, a new integrated estimation/guidance algorithm is introduced for the interception endgame of randomly maneuvering targets. The algorithm is based on a set of innovative concepts. First, separate estimator elements are used for the different tasks of target model identification, proper state estimation and jump detection. The joint contribution of the separate estimation elements enhances the accuracy of the state estimates at the critical moments of the interception. Second, explicit use of the time-to-go is made for scheduling the functions of the various estimators in a logic-based structure, depending on the detection of an eventual jump in the target maneuver command. Finally, two modifications are introduced in a perfect information differential game-based guidance law, thus alleviating the effect of the jump detection delay. One of the modifications (increased command gain) allows using the maximum available lateral acceleration of the interceptor at the end, whereas the other (using a dead zone) reduces the guidance error created during the period of the delay in the jump detection.

Table 2 Horizontal homing performance summary

Case	r_{av} m	r_{95} m	$p_{0.5}$
$\Delta t_d = 0$ s	0.095	0.22	0.996
$\Delta t_d = 0.05$ s	0.10	0.25	0.991
$\Delta t_d = 0.1$ s	0.23	1.47	0.885
$\Delta t_d = 0.1$ s, $k > 0$	0.15	0.69	0.931
$\Delta t_d = 0.1$ s, $k > 0$, DZ	0.10	0.30	0.981
Shinar et al. [20]	0.57	2.14	0.623

Using the new integrated logic-based estimation/guidance algorithm in a horizontal interception scenario example demonstrated not only a substantial improvement compared with earlier results, but also a potential to satisfy a hit-to-kill requirement. Validation of the results in a generic three-dimensional endoatmospheric ballistic missile defense scenario with time-varying parameters will be presented in a follow-up paper.

The crucial element for the successful application of the new algorithm is the existence of a sufficiently fast jump detector. The development of such a detector is currently under investigation.

The data used in the Monte Carlo simulations are generic and, therefore, the numerical results are only illustrative. However, the data represent a rather pessimistic case. In the example scenario, a nonexcessive interceptor maneuverability advantage ($\mu = 2$), a relatively agile target ($\varepsilon = 1$) and a conservative sensor noise ($\sigma_{ang} = 0.1$ mrad) were assumed. The bang–bang target maneuver, used in the simulations, is also the most efficient one for avoiding interception.

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