Differential-Game-Based Guidance Law using Target Orientation Observations

YAAKOV OSHMAN, Senior Member, IEEE

DAVID ARAD

Technion–Israel Institute of Technology

Modern 4th generation air-to-air missiles are quite capable of dealing with today's battlefield needs. Advanced aerodynamics, highly efficient warheads and smart target acquisition systems combine to yield higher missile lethality than ever. However, in order to intercept highly maneuverable targets, such as future unmanned combat air vehicles (UCAV), or to achieve higher tracking precision for missiles equipped with smaller warheads, further improvement in the missile guidance system is still needed. A new concept is presented here for deriving improved differential-game-based guidance laws that make use of information about the target orientation, which is acquired via an imaging seeker. The underlying idea is that of using measurements of the target attitude as a leading indicator of target acceleration. Knowledge of target attitude reduces the reachable set of target acceleration, facilitating the computation of an improved estimate of the zero-effort miss (ZEM) distance. In consequence, missile guidance accuracy is significantly improved. The new concept is applied in a horizontal interception scenario, where it is assumed that the target maneuver direction, constituting a partial attitude information, can be extracted via processing target images, acquired by an imaging sensor. The derivation results in a new guidance law that explicitly exploits the direction of the target acceleration. The performance of the new guidance law is studied via a computer simulation, which demonstrates its superiority over existing state-of-the-art differential-game-based guidance laws. It is demonstrated that a significant decrease in the miss distance can be expected via the use of partial target orientation information.

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Authors' current addresses: Y. Oshman, Dept. of Aerospace Engineering, Technion–Israel Institute of Technology, Haifa 32000, Israel, E-mail: (yaakov.oshman@technion.ac.il); D. Arad, Elbit Systems, Ltd., Haifa, Israel.

I. INTRODUCTION

Though employing state-of-the-art technology in other regards, most modern short range air-to-air missiles (SRAAM) still utilize variants of the classical proportional navigation (PN) guidance law [1]. This guidance law renders the missile normal acceleration proportional to the line-of-sight (LOS) rate, commonly measured by an infrared (IR) seeker. In [1] and [2] it was shown that a target can increase the miss distance against a PN-equipped SRAAM by performing hard evasive maneuvers. More advanced guidance laws, such as the augmented proportional navigation (APN) law, the optimal control guidance law (OGL) [1] or differential game-based laws [3–5] may very well achieve much smaller miss distances even against a maneuvering target. However, in addition to traditional LOS rate measurements, some of these laws require an estimate of the target acceleration at each and every moment as well.

Target acceleration can be estimated using LOS measurements, if a suitable model is assumed for the target dynamics [6, 7]. The common estimation method is based on the well-known Kalman filter (KF) or its extension to nonlinear systems, the extended Kalman filter (EKF). Another approach uses the multiple model adaptive estimator (MMAE) [8, 9] or its variants [10, 11], where the target acceleration is described by a number of models, each based on a different hypothesis about the target acceleration. The tasks of the estimator are, in this case, to identify the correct model (i.e., the model that best fits the true target acceleration) online, and to provide the guidance system with an accurate estimate of the target state. An application of the MMAE approach has recently appeared in [12], where it was used to deal with a complex scenario involving electronic countermeasures, used by the target to confuse the missile's RADAR seeker.

Although the closely related problems of target acceleration modeling and estimation have been investigated intensively over the last three decades, they still present significant challenges to missile designers. One of the major reasons to this is the fact that, in practical implementations, the estimation of target acceleration might become prohibitively slow in the presence of noise, rendering the filter's performance characterized by a considerable time lag, as well as by the poor quality of its estimate. In the critical endgame phase of the intercept, an excessively large time lag might result in an unacceptable miss distance, even if the estimator provides an otherwise excellent (but untimely) estimate of the target state. In particular, a maneuvering target is difficult to track because of the inherent time delay between the time of change of its acceleration, and the time when the trajectory of the target reflects this change unambiguously. Thus, when measurements are restricted to the point-mass properties of the target (e.g., LOS measurements), the filter's convergence must be significantly delayed.

To alleviate the estimation delay problem, a differential-game-based guidance law which partially compensates for the target estimation time lag has recently been proposed by Shinar and Shima [13]. Denoted DGL/C, this guidance law uses the center of the target maneuver reachable set as the estimated target instantaneous acceleration [14]. The reachable set is obtained under the assumption that the target maneuverability and the estimation time lag are known (or else can be closely approximated). In cases where this assumption holds, this approach has been shown to yield a lower maximal miss distance.

Motivated by [13] and expanding upon an earlier conference version [15], this paper exploits the correlation existing between the target's orientation and its evasion maneuvers to further advance the state-of-the-art in differential-game-based guidance laws. Thus, this work investigates the idea of enhancing the interception performance of an IR seeker-equipped SRAAM by utilizing information on the target orientation (namely, its attitude relative to a missile-fixed coordinate system), acquired in real-time by a computer vision algorithm processing data acquired by an imaging sensor installed onboard the missile's seeker.

Using imagery data to enhance target tracking performance has been investigated in the past decade in various contexts [16–21]. The interception of a highly maneuverable evading aircraft by a SRAAM has been recently addressed by the authors in [22]. In that work, it has been assumed that the missile's imaging seeker can acquire the target bank angle, when the target performs a horizontal turn maneuver. Using this information in a properly designed estimator, it has been shown that the missile's performance can be greatly enhanced due to improved target tracking.

Even if the target bank angle cannot be linked to its maneuver acceleration in an explicit, straightforward manner (e.g., if the target is not maneuvering strictly in the horizontal plane), an observation of the target bank angle can still reveal the current target maneuver direction. The current target maneuver direction can, in turn, be used to decrease the target acceleration reachable set, which, if properly used in a guidance law, should result in a smaller miss distance. This idea underlies the development of the new guidance law presented here.

The remainder of this paper is organized as follows. In Section II, existing differential-game-based guidance laws, namely, DGL/1 and DGL/C, are reviewed. The new guidance law is then derived. A numerical example is presented to demonstrate the performance of the new law and compare it with that of DGL/1 and DGL/C. Concluding remarks are offered in the final section.

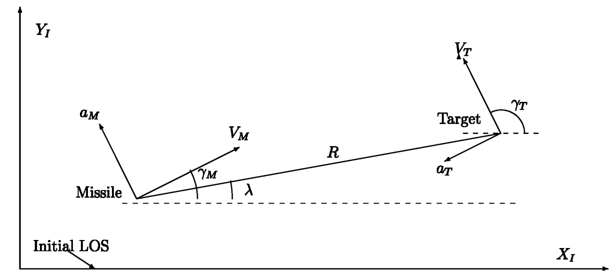

Fig. 1. Endgame scenario geometry.

II. DIFFERENTIAL GAME GUIDANCE LAWS

Although the interception problem is nonlinear, it is justifiable to perform a linear analysis about the initial LOS [1]. The coordinate system's origin is in the initial missile's center of gravity, its X_I axis is aligned with the initial LOS and its Y_I axis is perpendicular to it. The geometry of the endgame scenario is shown in Fig. 1, where λ is the LOS angle, γ_M and γ_T are the missile's and target's path angels, respectively, V_M and V_T are their respective velocities and a_M and a_T are their respective perpendicular accelerations. The slant range is denoted by R.

A. DGL/1

To set up a differential game between the missile (pursuer) and the target (evader), define the state vector

$$X = [y \ \ \dot{y} \ \ a_M \ \ a_T]^T \tag{1}$$

where y is the projection of the target position relative to missile along Y_I axis. Both the missile and the target are assumed to obey first-order dynamics with time constants τ_M and τ_T, respectively

$$\dot{a}_M = \frac{a_M^C - a_M}{\tau_M} \tag{2a}$$

$$\dot{a}_T = \frac{a_T^C - a_T}{\tau_T}. \tag{2b}$$

The maneuver commands are constrained according to

$$a_M^C \in [-a_M^{\max}, a_M^{\max}] \tag{3a}$$

$$a_T^C \in [-a_T^{\max}, a_T^{\max}] \tag{3b}$$

which, together with (2), renders the actual missile and target accelerations bounded as well.

The resulting state equation is

$$\dot{X} = \begin{bmatrix} x_2 \\ -x_3 + x_4 \\ -x_3/\tau_M + a_M^C/\tau_M \\ -x_4/\tau_T + a_T^C/\tau_T \end{bmatrix} \tag{4}$$

or

$$\dot{X} = AX + Ba_M^C + Ca_T^C \tag{5}$$

where the system matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1/\tau_M & 0 \\ 0 & 0 & 0 & -1/\tau_T \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ 0 \\ 1/\tau_M \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/\tau_T \end{bmatrix}. \tag{6}$$

The cost function for the differential game is set as

$$J = |y(t_f)| \tag{7}$$

or

$$J = |MX(t_f)|, \qquad M = [1\ \ 0\ \ 0\ \ 0]. \tag{8}$$

REMARK 1 The differential game can also be set up with cost functions involving quadratic functions of the state [5].

Since only $y(t_f)$ affects the cost function, an elegant order-reduction of the problem was suggested by Gutman [3] in the following manner.

The transition matrix associated with (5) is

$$\Phi(t,0) = \begin{bmatrix} 1 & t & -\tau_M^2\left(e^{-t/\tau_M} + \dfrac{t}{\tau_M} - 1\right) & \tau_T^2\left(e^{-t/\tau_T} + \dfrac{t}{\tau_T} - 1\right) \\ 0 & 1 & \tau_M(e^{-t/\tau_M} - 1) & -\tau_T(e^{-t/\tau_T} - 1) \\ 0 & 0 & e^{-t/\tau_M} & 0 \\ 0 & 0 & 0 & e^{-t/\tau_T} \end{bmatrix}. \tag{9}$$

Define the zero effort miss (ZEM) as the miss distance which will be achieved if neither of the players apply any maneuver command until the end of the game. At any time during the game, the ZEM can be computed as

$$Z(t) = M\Phi(t_f, t)X(t). \tag{10}$$

Using (9) and defining the time-to-go as

$$t_{go} \stackrel{\Delta}{=} t_f - t \tag{11}$$

the ZEM can be computed as

$$Z = y + \dot{y}t_{go} - a_M\tau_M^2\left(e^{-t_{go}/\tau_M} + \frac{t_{go}}{\tau_M} - 1\right) + a_T\tau_T^2\left(e^{-t_{go}/\tau_T} + \frac{t_{go}}{\tau_T} - 1\right). \tag{12}$$

Using (5) and (10) the ZEM can be shown to satisfy the following dynamics equation

$$\dot{Z} = -a_M\tau_M\left(e^{-t_{go}/\tau_M} + \frac{t_{go}}{\tau_M} - 1\right) + a_T\tau_T\left(e^{-t_{go}/\tau_T} + \frac{t_{go}}{\tau_T} - 1\right). \tag{13}$$

Since $Z(t_f) = y(t_f)$, (8) can be rewritten as

$$J = |Z(t_f)| \tag{14}$$

hence the Hamiltonian of the game is

$$\mathcal{H} = \lambda_Z\left[-a_M\tau_M\left(e^{-t_{go}/\tau_M} + \frac{t_{go}}{\tau_M} - 1\right) + a_T\tau_T\left(e^{-t_{go}/\tau_T} + \frac{t_{go}}{\tau_T} - 1\right)\right] \tag{15}$$

where λ_Z is the costate variable. Notice that $\tau(e^{-t_{go}/\tau} + (t_{go}/\tau) - 1)$ is always positive for any time constants and time-to-go (TGO).

The first-order necessary condition for optimality yields

$$\dot{\lambda}_Z = -\frac{\partial\mathcal{H}}{\partial Z} = 0 \tag{16}$$

and the transversality condition is

$$\lambda_Z(t_f) = \left.\frac{\partial J}{\partial Z}\right|_{t_f} = \mathrm{sgn}(Z(t_f)), \qquad Z(t_f) \neq 0. \tag{17}$$

Since λ_Z remains constant, the optimal strategies for both players are

$$a_T^* = \arg\max_{a_T}\mathcal{H} = a_T^{\max}\mathrm{sgn}(Z(t_f)) \tag{18a}$$

$$a_M^* = \arg\min_{a_M}\mathcal{H} = a_M^{\max}\mathrm{sgn}(Z(t_f)). \tag{18b}$$

Integrating (13) backwards from any final condition $Z(t_f)$ using (18) generates candidate optimal trajectories, from which it can be concluded that the game space comprises regions where a regular solution can be formulated in a state feedback form, and another (neutral) region, where the optimal strategies are arbitrary. This allows using the feedback form throughout the game space, yielding the DGL/1 guidance law [4]

$$a_M^C = a_M^{\max}\mathrm{sgn}(Z). \tag{19}$$

B. DGL/C

The DGL/1 guidance law was developed under the assumption of perfect information, i.e., it assumes that the target's instantaneous acceleration is perfectly known at any time. In practice, this is not true, since the target's acceleration has to be estimated by the missile from noisy measurements. This renders the target acceleration estimate both inaccurate and delayed, due to the inherent estimation time delay introduced by the missile's estimator. In turn, this degrades the computation of the ZEM (based on the target's acceleration estimate), resulting in large miss distances.

If it can be assumed that the estimation time delay, denoted by Δt, is perfectly known (or else can be assessed by the missile designer), the DGL/1 guidance law can be adapted to account for that delay, thus improving the guidance law's robustness [13]. In this case, the instantaneous target maneuver $a_T(t)$ is constrained, due to the target dynamics and bounded control, to the reachable set

$$a_T(t) \in [a_T(t)_{d\min}, a_T(t)_{d\max}] \tag{20}$$

where

$$a_T(t)_{d\min} = a_T(t-\Delta t)e^{-\Delta t/\tau_T} - a_T^{\max}(1 - e^{-\Delta t/\tau_T}) \tag{21a}$$

$$a_T(t)_{d\max} = a_T(t-\Delta t)e^{-\Delta t/\tau_T} + a_T^{\max}(1 - e^{-\Delta t/\tau_T}). \tag{21b}$$

The center of the target's maneuver reachable set is, therefore,

$$a_T(t)_{\text{center}} = a_T(t-\Delta t)e^{-\Delta t/\tau_T}. \tag{22}$$

Since the true target acceleration is unavailable to the missile, it cannot correctly compute the ZEM. However, using (22) in (12), as suggested by the intuitive approach of [13], an effective estimate of the ZEM (which is based on the center of the reachable set and not on the target's true instantaneous acceleration) can be computed as

$$\begin{aligned} Z_c = y + \dot{y}t_{\text{go}} - a_M\tau_M^2\left(e^{-t_{\text{go}}/\tau_M} + \frac{t_{\text{go}}}{\tau_M} - 1\right) \\ + a_T e^{-\Delta t/\tau_T}\tau_T^2\left(e^{-t_{\text{go}}/\tau_T} + \frac{t_{\text{go}}}{\tau_T} - 1\right). \end{aligned} \tag{23}$$

When used in (19), this corrected ZEM yields the DGL/C guidance law [13]. Shinar and Glizer [23] showed that (19), along with (23), constitute the optimal missile maneuver for the cost function (14).

III. NEW GUIDANCE LAW

A significant improvement of the DGL/C guidance law is possible, if one assumes that the target maneuver direction can be extracted via processing the target image, as acquired by the imaging sensor installed onboard the missile's seeker head. Such processing should make use of computer vision algorithms, which process the acquired sequence of 2-D target images to provide 3-D target attitude [24–28]. A fundamental assumption underlying this approach is that the target's attitude in 3-D space is correlated with its maneuver direction. This is the case with winged targets, that bank to turn, however, this approach will not be effective against radially symmetric targets (e.g., missiles) or targets that skid to turn.

Assuming that $\text{sgn}(a_T(t))$ is available from the missile's imaging seeker, the instantaneous target maneuver is constrained to the set $\mathcal{S}$, defined as

$$\mathcal{S} \overset{\Delta}{=} \begin{cases} [0,\infty] & \text{sgn}(a_T(t)) \geq 0 \\ [-\infty,0] & \text{sgn}(a_T(t)) < 0 \end{cases}. \tag{24}$$

However, the target maneuver is further bounded by its first-order dynamics and finite control. Hence the reachable set $\mathcal{S}_r$ is the intersection

$$\mathcal{S}_r = [a_T(t)_{d\min}, a_T(t)_{d\max}] \cap \mathcal{S}. \tag{25}$$

The center of the reachable set is now

$$a_T(t)_{\text{center}} = \begin{cases} a_T(t-\Delta t)e^{-\Delta t/\tau_T} & \text{sgn}(a_T(t)_{d\min}) = \text{sgn}(a_T(t)_{d\max}) \\ \frac{1}{2}a_T(t-\Delta t)e^{-\Delta t/\tau_T} + \frac{1}{2}\text{sgn}(a_T(t))a_T^{\max}(1-e^{-\Delta t/\tau_T}) & \\ & \text{sgn}(a_T(t)_{d\min}) \neq \text{sgn}(a_T(t)_{d\max}) \end{cases} \tag{26}$$

where $a_T(t-\Delta t)$ is available from the state estimator. Using the center of the reachable set to compute an effective estimate of the ZEM yields

$$\begin{aligned} Z_s = y + \dot{y}t_{\text{go}} - a_M\tau_M^2\left(e^{-t_{\text{go}}/\tau_M} + \frac{t_{\text{go}}}{\tau_M} - 1\right) \\ + a_T(t)_{\text{center}}\tau_T^2\left(e^{-t_{\text{go}}/\tau_T} + \frac{t_{\text{go}}}{\tau_T} - 1\right). \end{aligned} \tag{27}$$

The resulting missile's maneuver command is

$$a_M^C = a_M^{\max}\text{sgn}(Z_s) \tag{28}$$

which, together with (27), constitute the new guidance law. Since this guidance law utilizes the target's maneuver sign, it is termed DGL/S.

Discussion: 3-D Implementation: From the target's point of view, a planar, horizontal maneuver has two clear advantages: 1) the target does not lose kinetic energy while climbing, and 2) a negative-g dive is uncomfortable to the pilot (in the case of a manned target) and limited in intensity. Thus, it is reasonable to assume that planar, horizontal maneuvers can be used in many cases to model the target's evasive strategy with reasonable accuracy.

This paper focuses on presenting the concept of using target attitude to reduce its reachable set. To maintain this focus on the concept (and not on the details of its implementation), and based on the above arguments, the new guidance law has been derived assuming an interception scenario that takes place mainly in the horizontal plane. A comprehensive analysis of the 3-D interception problem is commonly done by addressing two decoupled planar solutions, namely, a horizontal solution (carried out in a plane perpendicular to Earth's gravity direction) and a

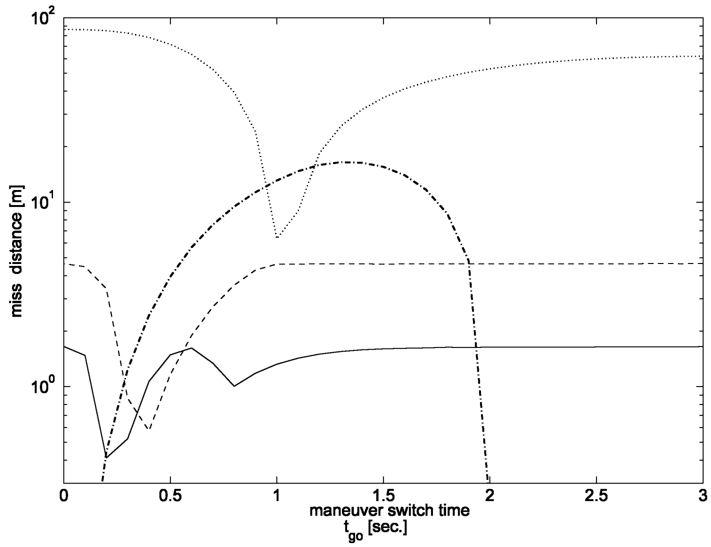

Fig. 2. Miss distance versus target maneuver direction switch TGO. Continuous line: DGL/S, dashed line: DGL/C, dash-dotted line: DGL/1, dotted line: PN.

vertical solution. The total miss distance is then the root sum square of the horizontal and vertical misses. To implement the new concept presented herein in a 3-D interception scenario, it should be complemented by a suitable law designed for the vertical plane. Since the target maneuver in the vertical plane is expected to be moderate, DGL/C or even the PN guidance law can be used. However, an implementation of the attitude-based concept presented herein in the vertical plane should not be completely ruled out: if a violent vertical maneuver is detected, its direction might also be extracted via processing the imaging seeker data (by distinguishing between "nose up" and "nose down" image states). In such cases, a vertical version of DGL/S can be used to further reduce the target reachable set. Furthermore, contrary to the horizontal plane, in the vertical plane an asymmetrical law can be derived that takes into account the asymmetry between "up" and "down" maneuvers.

IV. NUMERICAL EXAMPLE

A head-on scenario was chosen for the numerical example. The performance of the three guidance laws described in this paper was examined in a linear deterministic simulation. In this scenario the missile maneuverability is limited to 20 g. The target performs a violent bang-bang maneuver, which is known to be the optimal evasion maneuver in perfect-information differential games [3, 4]. The target's maneuver maximum magnitude is 12 g (i.e., the maneuver starts with a 12 g command to one side and switches abruptly, at a certain instant, to a 12 g command to the other side). Although a 12 g maneuver is beyond the physical capability of a human pilot, it is well inside the feasible envelope of a UCAV.

TABLE I
Linear Simulation Parameters

Simulation Parameter	Value
R_0 [m]	3,000
λ_0 [rad]	0
V_M [m/s]	700
V_T [m/s]	300
γ_{M0} [rad]	0
γ_{T0} [rad]	π
τ_M [s]	0.3
τ_T [s]	0.2
Δt [s]	0.4

The remaining simulation parameters are listed in Table I, where a subscript zero denotes a value at the beginning of the game.

A. Guidance Laws Comparison

The behavior of the miss distance as a function of the target maneuver direction reversal time (measured from the end of the game) is shown in Fig. 2. The guidance laws compared are the three differential game based laws, along with the well-known PN guidance law, implemented as

$$a_M^C = N' V_c \dot{\lambda} \qquad (29)$$

where V_c is the closing velocity and N' is the PN constant (set to 4 in this study). The PN law is included in this comparison because it might be argued that it is less susceptible to the estimation time delay since it does not explicitly utilize the target acceleration. Nevertheless, as Fig. 2 shows, the performance of the PN law is much worse than the performance of the differential game based laws, and,

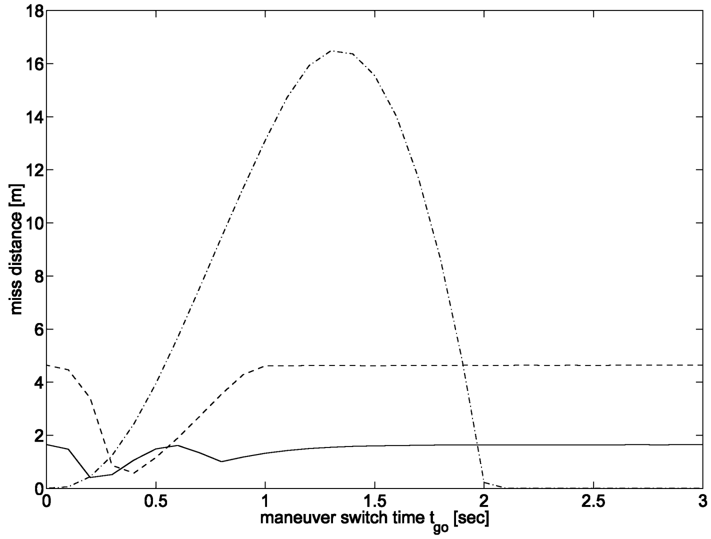

Fig. 3. Miss distance versus target maneuver direction switch TGO. Continuous line: DGL/S, dashed line: DGL/C, dash-dotted line: DGL/1.

TABLE II
Maximum and Average Miss Distance

Guidance Law	Average Miss Distance [m]	Maximum Miss Distance [m]
DGL/1	5.62	16.48
DGL/C	4.29	4.64
DGL/S	1.53	1.65

as shown in Figs. 7 and 8 in the sequel, its inferiority relative to the DGL/C and DGL/S laws remains valid even when these laws are implemented with a highly inaccurate estimation time delay.

To closely observe the behavior of the differential-game-based laws, their performance is shown again in Fig. 3 (the PN law is omitted to allow linear scaling of the miss distance axis). It is obvious that using DGL/1 yields unacceptably large miss distances if the maneuver direction change occurs shortly before the interception ends. On the other hand, using DGL/C or DGL/S renders the miss distance quite robust with respect to the maneuver direction reversal time, and, moreover, the resulting miss distances are much lower than the maximum miss distance corresponding to DGL/1. Table II shows a 64% decrease in the maximum miss distance with respect to DGL/C and a similar decrease in the average miss distance. DGL/1 exhibits poor performance and is clearly not competitive with both DGL/C and DGL/S guidance laws.

The maximum miss distance for missiles equipped with DGL/C and DGL/S is shown in Fig. 4 versus the missile's estimator time delay. As could be expected, for both laws the maximum miss distance increases as the time delay grows, since the reachable set increases, rendering the ZEM estimate less effective. This behavior remains true for both guidance laws up to a point (different for both laws) where the reachable set no longer grows with the estimation time delay. For the DGL/C law this point is reached asymptotically, and is determined by the target's acceleration limits, that is, the limiting reachable set is equal to the feasible acceleration set $[-a_T^{\max}, a_T^{\max}]$. In this case, when the estimation time delay is large enough so that the limiting reachable set is nearly reached, the target acceleration estimate becomes totally useless and the performance of the DGL/C law degrades to that of a guidance law that does not use any explicit information on the target's acceleration (this law is called DGL/0 in the literature). On the other hand, in the case of the DGL/S law the limiting reachable set is determined both by the target feasible acceleration limits and by the additional information on the target maneuver direction, obtained from the imaging seeker measurements. Hence, in this case the reachable set never grows to equal the feasible acceleration set, which explains the clear superiority of DGL/S over DGL/C for large estimation delays, as evidenced from Fig. 4.

Notice also that, because a bang-bang target maneuver was selected for this example, for small estimation time delays the maximum miss distances are equal for both laws. This can be explained by noting that when the target maneuvers at maximum acceleration, and assuming a small estimation time delay, the symmetric reachable set computed by the DGL/C law (which does not utilize the maneuver direction information) is identical with the asymmetric reachable set computed by the DGL/S law (which does take into account the maneuver direction measurement), since $\mathrm{sgn}(a_T(t)_{d\min}) = \mathrm{sgn}(a_T(t)_{d\max})$. Thus, the centers of both reachable sets are identical, yielding identical ZEM estimates for both guidance laws, which results in identical miss distances.

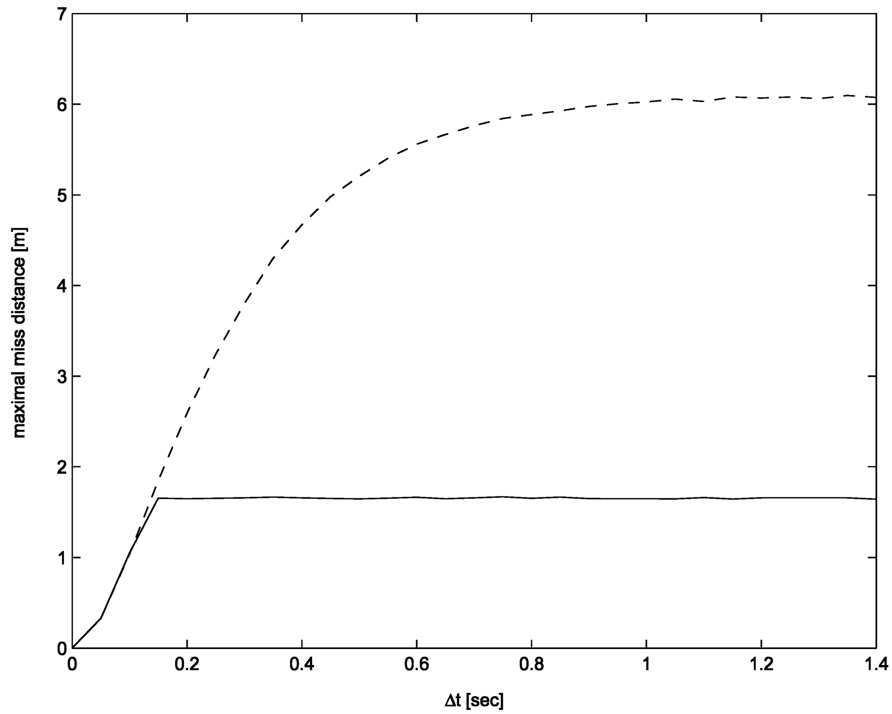

Fig. 4. Maximum miss distance versus estimation time lag. Continuous line: DGL/S, dashed line: DGL/C.

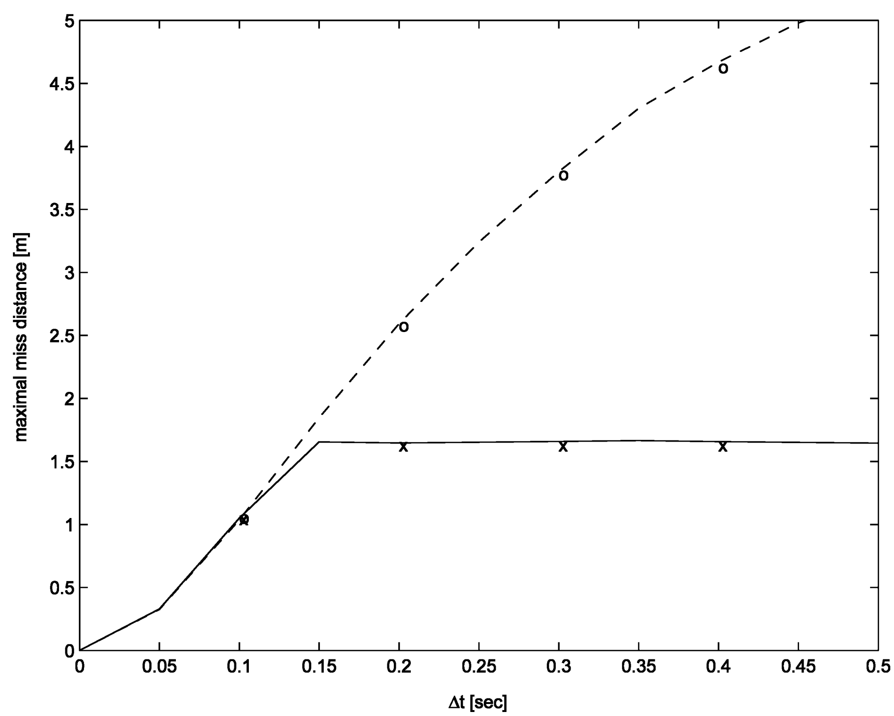

Fig. 5. Nonlinear simulation correlation: maximum miss distance versus estimation time lag. Continuous line: DGL/S linear simulation, dashed line: DGL/C linear simulation, "x"s: DGL/S nonlinear simulation, circles: DGL/C nonlinear simulation.

Some of the linear simulation results were verified against a nonlinear simulation. The correlation between the linear and nonlinear simulations is shown in Fig. 5, which demonstrates excellent agreement between the two simulations, thus substantiating the assumption underlying the linear analysis of the preceding section.

B. Uncertain Target Acceleration

The implementation of DGL/S requires knowledge of the maximum target maneuver command. To assess the sensitivity of the new guidance law with respect to uncertainty in this parameter, a numerical simulation study was performed. Fig. 6 shows the miss distances

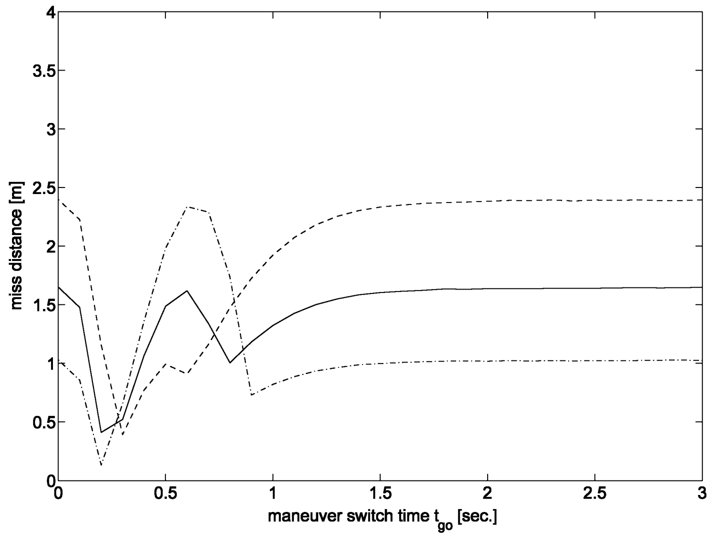

Fig. 6. Miss distance versus target maneuver direction switch TGO for uncertain maximum target maneuver. Continuous line: 12 g (exact) maximum maneuver, dashed line: 9 g (underestimated) maneuver, dash-dotted line: 15 g (overestimated) maneuver.

TABLE III
Maximum and Average Miss Distance for Uncertain Maximum Target Maneuver

Assumed Maximum Maneuver	Average Miss Distance [m]	Maximum Miss Distance [m]
12 g (True)	1.53	1.65
9 g	2.16	2.40
15 g	1.07	2.34

obtained when the DGL/S law is used with exact maximum target maneuver acceleration (12 g), underestimated acceleration (9 g), and overestimated acceleration (15 g). As could be expected, when the target's acceleration capability is underestimated by the guidance law, the resulting miss distances are generally larger than the nominal miss distances obtained for the exact maximum maneuver. Similarly, an overestimated target's maneuver capability results, generally, in lower miss distances. However, while the guidance law's performance is robust for both the exact and underestimated target's maximum maneuver, in the sense that it can only improve (if the target performs nonoptimal evasive maneuvers), the situation is reversed when the target's maximum maneuver is overestimated, in which case certain target maneuvers (close to the end of the scenario) can generate significantly larger miss distances.

Summarized in Table III, the results indicate that even in the case of extremely inaccurate target maximum maneuver, the guidance law's performance is still superior with respect to that of both DGL/1 and DGL/C.

C. Estimation Delay Uncertainty

The performance of the DGL/C and DGL/S laws assumes knowledge of the estimation time delay. To evaluate the sensitivity of these laws to errors in this variable, additional simulations were carried out where these laws were implemented with erroneous time delays.

Fig. 7 depicts the miss distances obtained when the DGL/C law is used with exact estimation time delay (0.4 s), underestimated delay (0.3 s), and overestimated delay (0.5 s). In general, it is clear that the guidance law's performance is not severely affected by the 25% estimation delay errors. It is interesting to note that the minimal miss distance associated with the underestimated time delay is somewhat lower than that obtained using the exact time delay. Nevertheless, as could be expected, the maximal miss distance (which is the parameter used to assess the single shot kill probability) is minimal for the exact time delay.

Fig. 8 shows the behavior of the DGL/S law, when $\pm 25\%$ estimation delay errors are implemented. As can be observed, the DGL/S law is significantly less susceptible to estimation delay errors than the DGL/C law. This behavior could be expected, as the DGL/S law enjoys the availability of the instantaneous maneuver sign information, which is not affected by the estimation delay errors. Although partial, this additional information serves to render the DGL/S law more robust with respect to estimation delay uncertainty relative to the DGL/C law.

V. CONCLUSIONS

This paper presents a new concept in advanced missile guidance theory. The concept is based on the idea of using target attitude information for deriving improved differential-game-based guidance laws. Information on target attitude can be acquired from imaging sensors, installed onboard advanced missile seekers. Target attitude measurements can be used as a

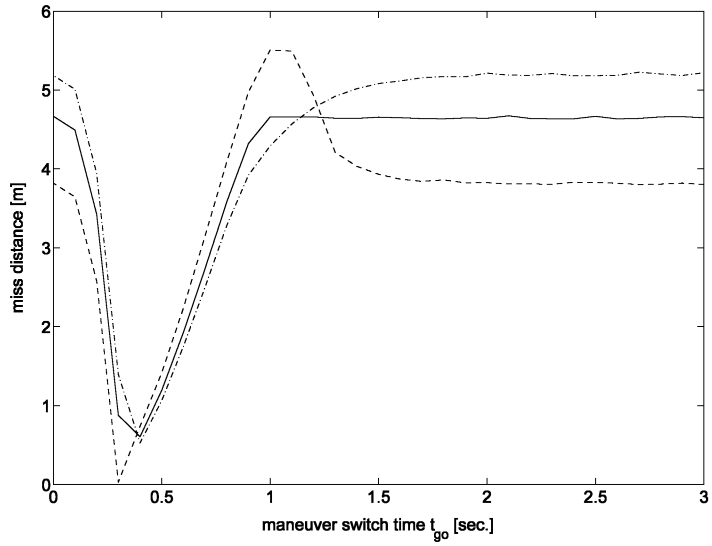

Fig. 7. Miss distance versus target maneuver direction switch TGO for uncertain target maneuver estimation delay for DGL/C. Continuous line: 0.4 s (exact), dashed line: 0.3 s (underestimated), dash-dotted line: 0.5 s (overestimated) time delay.

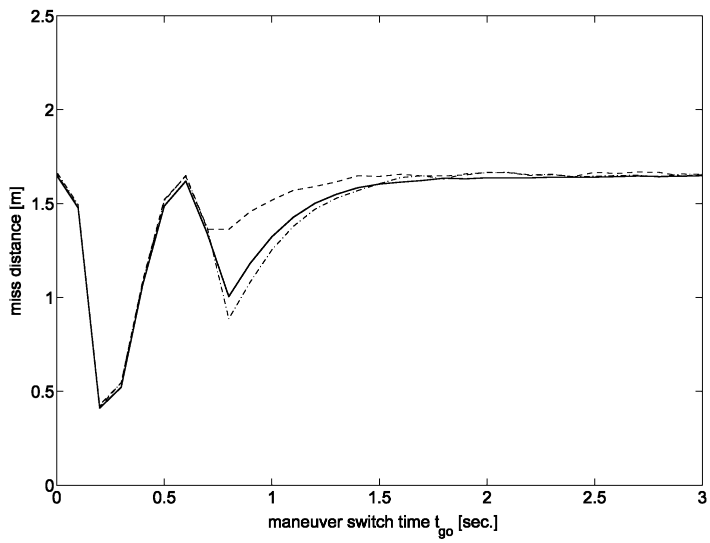

Fig. 8. Miss distance versus target maneuver direction switch TGO for uncertain target maneuver estimation delay for DGL/S. Continuous line: 0.4 s (exact), dashed line: 0.3 s (underestimated), dash-dotted line: 0.5 s (overestimated) time delay.

leading indicator of target acceleration, thus reducing the target acceleration reachable set. This facilitates the computation of an improved estimate of the ZEM distance, which, in turn, translates to significantly enhanced missile guidance accuracy.

The new concept is used to derive a differential-game-based guidance law for short-range air-to-air horizontal plane interception scenarios. This law makes use of the target acceleration direction, a partial target orientation information. Such information can be obtained from a missile imaging seeker, in the case of a winged target (that banks to turn). Though being incomplete, this information is efficiently used to decrease the target maneuver reachable set and, thereby, enhance the guidance performance. A simulation study is used to compare the new guidance law, in a head-on interception scenario, to three well-known guidance laws (differential-game based DGL/1 and DGL/C, and the popular PN) via a numerical simulation study. Regarding the miss distance as the figure of merit, it is shown that a substantial performance improvement is achieved, even relative to the recently proposed DGL/C law, that explicitly takes into account the time delay of the missile's estimator.

The idea presented herein complements a previous work by the authors, where the target orientation information has been used to enhance the performance of the missile's tracking algorithm. Thus, the combination of both works constitutes a unified approach to modern, imaging-based, missile tracking and guidance. Both components of this approach, namely, the target estimator and the guidance law, are based on the availability of target attitude information.

The integration of these algorithms with a computer vision algorithm whose purpose is to extract the target attitude information from the acquired imaging data is a topic of current research.

It should be noted that target orientation information is not meant to replace conventional LOS measurements. Rather, the new measurement should augment all previously available conventional information. Moreover, it is plausible that the proposed concept can be implemented in currently operational missiles by introducing relatively minor software changes in the missile's guidance system.

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Yaakov Oshman (AM'96—SM'97) received his B.Sc. (summa cum laude) and D.Sc. degrees, both in aeronautical engineering, from the Technion–Israel Institute of Technology, Haifa, Israel, in 1975 and 1986, respectively.

From 1975 to 1981 he was with the Israeli Air Force, where he worked in the areas of structural dynamics and flutter analysis and flight testing. In 1987 he was a research associate at the Department of Mechanical and Aerospace Engineering of the State University of New York at Buffalo, where he was, in 1988, a visiting professor. Since 1989 he has been with the Department of Aerospace Engineering at the Technion–Israel Institute of Technology, where he is currently a professor. During the 1996/1997 and 1997/1998 academic years he spent a sabbatical with the Guidance, Navigation and Control Center of NASA's Goddard Space Flight Center, where he worked in research and development of spacecraft attitude estimation algorithms. He has consulted to RADA Electronic Industries, Ltd., RAFAEL Ltd., and the Israeli Ministry of Defense. His research interests are in advanced optimal estimation, information fusion, and control methods and their application in aerospace guidance, navigation, and control systems. Of particular interest are interdisciplinary aerospace systems, including structural estimation and control, and health monitoring/fault detection and isolation systems.

Dr. Oshman is the President of the Israeli Association for Automatic Control, a national member organization of the International Federation of Automatic Control (IFAC). He headed the Philadelphia Flight Control Laboratory at the Technion (1993–1996). He published over 120 journal and conference papers and book chapters. He is a coauthor of the paper that was awarded the Best Paper Award of the 2002 AIAA Astrodynamics Specialist Conference, and a coauthor and advisor of the paper that has been awarded the Best Paper Award of the 2004 AIAA Guidance, Navigation and Control Conference. He has received the Technion's Raymond and Miriam Klein Research Prize for his research on enhanced air-to-air missile tracking using target orientation observations (2002), and the Technion's Meir Hanin Research Prize for his work on spacecraft angular velocity estimation (2004). The latter work has been put to use in the Israeli AMOS-2 communication satellite. He has been on the program committees of over a dozen international conferences. He is an editor for Guidance and Control Systems for the *IEEE Transactions on Aerospace and Electronic Systems*, and a member of the editorial board of the AIAA *Journal of Guidance, Control and Dynamics*. He is an Associate Fellow of the AIAA.

David Arad received his B.Sc. (cum laude) and M.Sc. (cum laude) degrees, both in aerospoace engineering, from the Technion–Israel Institute of Technology, Haifa, Israel, in 1997 and 2002, respectively.

He is currently a technical manager at Elbit Systems Ltd., Haifa, Israel, where he is responsible for developing avionics for airborne systems.