Optimal Sensor Selection Strategy for Discrete-Time State Estimators

YAAKOV OSHMAN
Technion-Israel Institute of Technology

A novel sensor selection strategy is introduced, which can be implemented on-line in time-varying discrete-time systems. We consider a case in which several measurement subsystems are available, each of which may be used to drive a state estimation algorithm. However, due to practical implementation constraints (such as the ability of the on-board computer to process the acquired data), only one of these subsystems can actually by utilized at a measurement update. An algorithm is needed, by which the optimal measurement subsystem to be used is selected at each sensor selection epoch. The approach taken here at solving this problem is based on using the square root V-Lambda information filter as the underlying state estimation algorithm. This algorithm continuously provides its user with the spectral factors of the estimation error covariance matrix, which are used in this work as the basis for an on-line decision procedure by which the optimal measurement strategy is derived. At each sensor selection epoch, a measurement subsystem is selected, which contributes the largest amount of information along the principal state space direction associated with the largest current estimation error. A numerical example is presented, which demonstrates the performance of the new algorithm. The state estimation problem is solved for a third-order time-varying system equipped with three measurement subsystems, only one of which can be used at a measurement update. It is shown that the optimal measurement strategy algorithm enhances the estimator by substantially reducing the maximal estimation error.

Manuscript received January 6, 1991; revised December 15, 1992.

IEEE Log No. T-AES/30/2/15478.

The research was supported by Technion V.P.R. Fund, L. Kraus Research Fund.

Author's address: Technion–Israel Institute of Technology, Aerospace Engineering, Technion City, Haifa 32000, Israel.

0018-9251/94/$4.00 © 1994 IEEE

I. INTRODUCTION

In many engineering situations, a variety of possible measurements can be carried out on a dynamic system, whose state is of interest. These measurements may serve to drive a computational state estimation scheme (like the Kalman filter or its variants). For example, in large space structures (LSS), it is possible to use cheap, easy to mount piezoelectric sensors in a highly distributed fashion in many (or all) components of the flexible structure. Because LSS are continuum structures which require large dimensional models, a highly distributed measurement system may be of a great importance, especially when stringent requirements of shape, orientation, vibration suppression and pointing accuracy are imposed. However, in practice, some physical constraints will prohibit the usage of such a system to its full capability, i.e., only a limited subset of all sensors installed in the structure could actually be used at each measurement epoch. Thus, for example, the computational burden associated with the on-line processing of too many measurements may require computing resources that may not be available (taking into consideration the fact that the on-board computer will, most probably, be busy with other tasks, such as computing control gains, etc.). In some other instances, when the measured signals have to be transmitted to the processing unit using a (possibly time-shared) communication link, this link may have band limitations that may dictate the communication of only a limited set of all the information acquired. Thus, it is seen that in some cases the question may arise as to which subset of measurements, out of all measurements available at a particular time, should be used by the on-board computer to drive the estimation algorithm, such that a certain optimality criterion is satisfied. The research reported herein is involved with this question.

The dynamic system considered here is modeled by a lumped-parameter time-varying, discrete-time mathematical model. The measurement system consists of a set of discrete-time subsytems, not all of which can be used at the same time. It is required that the algorithm for determining the optimal sensor selection strategy be an on-line algorithm, in order to accommodate systems whose time-varying parameters cannot be predicted very reliably before the mission (e.g., LSSs, whose in-space behavior cannot be assessed accurately based on very limited on-Earth testing). The criterion for determining the optimal measurement strategy is derived out of optimum state estimation requirement.

The optimal measurement strategy problem has been addressed in the past by several authors, although none of them has dealt with the case of on-line strategy determination. In [3, 11], the case of measurement subsystems, that had costs associated with their usage at each time, was considered. The problem was formulated as a general optimization problem with a cost functional which involved the trace of the estimation error covariance matrix, and was solved using the matrix minimum principle. This led to the formulation of a complex, nonlinear, two-point boundary value problem, which can be solved practically only off-line using methods such as Kelley's min-H technique [12]. This, in turn, means that the method is not suitable for use in time-varying problems, in which the grounds for determining the optimal measurement policy are not known in advance. Meier, et al. [15] treated the control of measurement subsystems within a stochastic feedback control system, using a dynamic programming approach. This approach yielded a nonlinear, deterministic, optimal control problem. A somewhat related problem of optimally allocating sensors in a distributed-parameter system was treated in [1, 6, 18], however, in these cases an off-line solution for the (time-invariant) optimal location of sensors in the system was determined.

A different approach is taken here, which alleviates the computational burden problem and facilitates the implementation of the algorithm in on-line applications. The proposed method is based on the square-root (SR) solution of the estimation problem using the covariance spectral factors [17]. Most commonly, the main motivation for using an SR estimator stems from its excellent numerical characteristics, which are superior to those of the conventional Kalman filtering algorithm. In our case, however, another motivation for using a special SR formulation (based on the spectral decomposition of the estimation error covariance matrix), called the V-Lambda algorithm, stems from the fact that this SR algorithm provides the spectral factors of the estimation error covariance continuously as the estimation process evolves in time. Thus, the user is provided with an invaluable physical insight into the estimation process [10]. Using the information obtained from the spectral factors, a decision algorithm may be derived, by which the measurement strategy problem can be easily solved using the on-board computer in an on-line fashion.

In the next section the mathematical model of the discrete-time system under consideration is defined, and the optimal measurement strategy problem is formulated within the framework of the given mathematical model. Since the measurement strategy is determined using the V-Lambda SR filtering algorithm as the underlying estimation technique, this algorithm is detailed for completeness in Section III. The optimal measurement strategy algorithm is next derived in Section IV, followed by a numerical example in Section V. Some concluding remarks are offered in Section VI.

II. PROBLEM FORMULATION

In the ensuing, the following notation is used to describe the discrete-time stochastic process whose state is to be estimated:

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + G_k \mathbf{w}_k. \tag{1}$$

Here $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector, $\{\mathbf{w}_k\} \in \mathbb{R}^p$ is a Gaussian white sequence with zero mean and positive definite covariance Q_k, and $F_k \in \mathbb{R}^{n,n}$ is the state transition matrix. It is assumed that the initial condition random vector $\mathbf{x}_0$ has a Gaussian distribution with mean μ and positive definite covariance P_0.

The state of the system can be observed through any one of the following M measurement subsystems:

$$\mathbf{y}_k^j = H_k^j \mathbf{x}_k + \mathbf{v}_k^j, \qquad j = 1,2,\ldots,M \tag{2}$$

where $\mathbf{y}_k^j \in \mathbb{R}^{m_j}$ is the measurement vector of the jth measurement subsystem at time t_k, $\{\mathbf{v}_k^j\} \in \mathbb{R}^{m_j}$ is the Gaussian white noise sequence of the jth measurement subsystem with zero mean and positive definite covariance R_k^j, and $H_k^j \in R^{m_j}$ is the measurement matrix associated with the jth measurement subsystem. It is further assumed that the measurement noise (associated with each of the measurement subsystems), the process noise, and the initial condition random vector are not correlated.

Having defined the mathematical model of the system under consideration, we now state the optimal measurement configuration strategy problem which is addressed in this work.

A. Problem Statement

Suppose that at each measurement update epoch only one measurement subsystem can be utilized (because of physical constraints such as those mentioned in the Introduction). The problem is to determine an optimal sensor selection strategy (a sequence $\{\hat{j}(t_k)\}$), which specifies which measurement subsystem (designated by its serial number $\hat{j}$) is to be used at the measurement update epoch t_k, such that a certain estimation optimality criterion is satisfied. Before presenting the optimality criterion chosen, the following well-known facts from estimation theory are recalled [7].

Since we are dealing with a linear dynamic system whose inputs (the process and measurement noises), as well as the initial state random vector, are assumed to be Gaussian distributed, the probability density function (pdf) of the state estimate (conditioned on the measurement history) is also Gaussian [14]. This pdf is characterized by a quadratic form involving the estimation error and the error covariance matrix; thus, if the estimation error at t_k given the measurements up to and including t_k is defined as

$$\tilde{\mathbf{x}}_{k/j} = \mathbf{x}_k - \hat{\mathbf{x}}_{k/j}$$

and the associated error covariance matrix is $P_{k/j}$, this quadratic form is

$$Q_{k/j} = \tilde{\mathbf{x}}_{k/j}^T P_{k/j}^{-1} \tilde{\mathbf{x}}_{k/j}$$

and $Q_{k/j}$ can be used to calculate an equi-error ellipsoid in state space, which describes the geometrical distribution of the "quality" of the state estimate; the semiaxes of the error ellipsoid have lengths equal to the square roots of the eigenvalues of the covariance matrix, and the orientations of these axes (the principal axes) in state space are given by the covariance matrix eigenvectors. Thus, if $V_{k/j}$ is the matrix formed by taking the eigenvectors of the covariance $P_{k/j}$ as its columns, and the following linear transformation is defined

$$\mathbf{x}_k := V_{k/j} \mathbf{z}_k$$

then the estimation error covariance matrix corresponding to the transformed vector $\mathbf{z}_k$ is the diagonal matrix of the eigenvalues of $P_{k/j}$ (the error covariance of $\mathbf{x}_k$). Therefore, each eigenvalue of the covariance matrix is an estimation error variance along a principal direction in state space, which is uncorrelated with all other principal directions and is defined by the corresponding eigenvector (i.e., this principal direction is one of the axes of the error ellipsoid). Moreover, the eigenvector corresponding to the largest eigenvalue of the covariance matrix (whose value is the length of the largest semiaxis of the error ellipsoid) defines the direction in state space along which the quality of estimation is the poorest.

With these basic ideas on hand, the optimality criterion which is used in the sequel is stated next.

B. Optimality Criterion

At each sensor selection epoch, the objective of the proposed algorithm is to maximize the amount of information (contributed to the system by the measurement) along the state space direction associated with the maximal estimation error.

In light of the preceding discussion, the meaning of the optimality criterion as stated above is that the measurement system is chosen according to its information contribution along the state space direction defined by the eigenvector (or, more generally, eigenvectors) corresponding to the largest eigenvalue. Since the solution to the problem can be most naturally presented within the framework of the V-Lambda SR filter, this estimation algorithm is outlined next.

III. DISCRETE-TIME V-LAMBDA ESTIMATION ALGORITHM

In this section the discrete-time (information mode) V-Lambda estimation algorithm is described (for derivation see [17]). Complying with the conventional discrete-time Kalman filter style, the algorithm consists of two subalgorithms, namely, the measurement-update (which incorporates the new information contained in the most recently acquired measurement into the estimation process), and the time-update algorithm (by which the state estimates, along with the estimation error covariance matrix, are propagated in time between measurement epochs). However, contrary to the Kalman filter (and similarly to other SR algorithms), the covariance matrix is replaced in the V-Lambda algorithm by its factors, which are chosen, in this specific case, to be the spectral factors: V, the matrix whose columns are the covariance eigenvectors, and Λ, the diagonal matrix whose entries are the covariance eigenvalues (other SR estimation algorithms are based on other SR factorizations of the covariance, such as the U-D factorization [5] and the QR factorization [8]). Both update levels of the algorithm are presented below.

A. V-Lambda Measurement Update Level

The measurement update problem is as follows. Given the SR spectral factors $V_{k/k-1}$ and $\Lambda_{k/k-1}^{-1/2}$ of the a-priori information matrix $P_{k/k-1}^{-1}$ at t_k, where $P_{k/k-1}$ is the a-priori estimation error covariance, $V_{k/k-1}$ is the eigenvector matrix, $\Lambda_{k/k-1}$ is the diagonal eigenvalue matrix and $P_{k/k-1} = V_{k/k-1}\Lambda_{k/k-1}V_{k/k-1}^T$, and given the a-priori normalized state estimate $\hat{\mathbf{d}}_{k/k-1}$ (defined below in (4)), compute the a-posteriori SR factors $V_{k/k}$ and $\Lambda_{k/k}^{-1/2}$, and the updated normalized estimate $\hat{\mathbf{d}}_{k/k}$, defined as

$$\hat{\mathbf{d}}_{k/k} := \Lambda_{k/k}^{-1/2} V_{k/k}^T \hat{\mathbf{x}}_{k/k}. \tag{3}$$

Before stating the measurement update algorithm, it must be noted that, since in our case there exist M measurement subsystems from which we can choose only one at each measurement update epoch (or, more generally, at each sensor selection epoch), the results of the measurement update will vary according to which subsystem was active at the particular update epoch. However, for each measurement subsystem, predetermined before the actual update takes place, the measurement update algorithm described below is the optimal (minimum variance, unbiased) algorithm which is algebraically equivalent to the Kalman filter update. We assume, therefore, that the jth measurement subsystem was chosen to be active in the measurement update at time t_k. The actual measurement update algorithm is summarized below.

Given the time propagated factors $V_{k/k-1}$ and $\Lambda_{k/k-1}^{-1/2}$, the nonsingular measurement noise covariance R_k and the a-priori normalized estimate $\hat{\mathbf{d}}_{k/k-1}$, where

$$\hat{\mathbf{d}}_{k/k-1} := \Lambda_{k/k-1}^{-1/2} V_{k/k-1}^T \hat{\mathbf{x}}_{k/k-1} \tag{4}$$

and assuming that the jth measurement subsystem is active, define an augmented matrix $A_k \in \mathbb{R}^{n+m,n}$ as

$$A_k := \begin{bmatrix} \Lambda_{k/k-1}^{-1/2} V_{k/k-1}^T \\ R_k^{j^{-1/2}} H_k^j \end{bmatrix} \tag{5}$$

and perform a singular value decomposition (SVD) of it to obtain

$$A_k = Y_k \begin{bmatrix} \Sigma_k \\ 0 \end{bmatrix} U_k^T. \tag{6}$$

Then, the measurement updated spectral factors are related to the SVD factors of A_k as follows

$$V_{k/k} = U_k \tag{7a}$$

$$\Lambda_{k/k}^{-1/2} = \Sigma_k. \tag{7b}$$

Moreover, define $\mathbf{b}_k$ as

$$\mathbf{b}_k = \begin{bmatrix} \hat{\mathbf{d}}_{k/k-1} \\ R_k^{j^{-1/2}} \mathbf{y}_k^j \end{bmatrix} \tag{8}$$

and premultiply it by Y_k^T; then, partitioning the resulting vector in accordance with the partition of $\mathbf{b}_k$, the updated normalized estimate is found as follows

$$Y_k^T \mathbf{b}_k = \begin{bmatrix} \hat{\mathbf{d}}_{k/k} \\ \mathbf{e}_k \end{bmatrix} \tag{9}$$

where the m_j-vector $\mathbf{e}_k$ is the estimation residual (the innovation) [17]. (Note: $Y_k \in \mathbb{R}^{n+m_j,n+m_j}$ and $U_k \in \mathbb{R}^{n,n}$ are the orthogonal matrices of the left and right singular vectors of A_k, respectively, and $\Sigma_k \in \mathbb{R}^{n,n}$ is a diagonal matrix whose non-zero elements are the singular values of A_k). The time-update algorithm is independent of the particular measurement subsystem used, and is therefore the ordinary information mode V-Lambda time-update, summarized below.

B. V-Lambda Time-Update Level

Given the a-posteriori SR information factors $V_{k/k}$ and $\Lambda_{k/k}^{-1/2}$, where $P_{k/k} = V_{k/k}\Lambda_{k/k}V_{k/k}^T$, the nonsingular transition matrix F_k, the input gain matrix $G_k \in \mathbb{R}^{n,p}$ and the nonsingular process noise covariance $Q_k \in \mathbb{R}^{p,p}$, define the augmented array $B_k \in \mathbb{R}^{n+p,n+p}$:

$$B_k := \begin{bmatrix} Q_k^{-1/2} & 0 \\ \Lambda_{k/k}^{-1/2} V_{k/k}^T F_k^{-1} G_k & \Lambda_{k/k}^{-1/2} V_{k/k}^T F_k^{-1} \end{bmatrix} \tag{10}$$

and perform a partial triangularization of it; that is, find an orthogonal transformation τ such that

$$\tau B_k = \begin{bmatrix} M_k & L_k \\ 0 & N_k \end{bmatrix} \tag{11}$$

where $M_k \in \mathbb{R}^{p,p}$ is upper triangular. Proceed with an SVD of N_k in (11) to obtain

$$N_k = W_k S_k Z_k^T \tag{12}$$

where W_k, Z_k are the orthogonal matrices of the left and right singular vectors of N_k, respectively, and S_k is the diagonal singular values matrix; then, the a-priori eigenfactors at t_{k+1} are given by

$$\Lambda_{k+1/k}^{-1/2} = S_k \tag{13a}$$

$$V_{k+1/k} = Z_k \tag{13b}$$

and the time propagation of the state estimate is performed according to

$$\hat{\mathbf{x}}_{k+1/k} = F_k \hat{\mathbf{x}}_{k/k} \tag{14}$$

(which is the regular Kalman filter algorithm).

In the next section we show how the V-Lambda estimation algorithm is utilized in a natural manner as a means for determining the required optimal measurement policy.

IV. MEASUREMENT CONFIGURATION USING INFORMATION CONCEPTS

Reviewing the V-Lambda filtering algorithm presented in the preceding section, it is clear that since the eigenfactors (the eigenvalues and eigenvectors) of the estimation error covariance matrix are serving as the estimation variables (thus replacing the covariance matrix itself), they are available to the user at each moment during the estimation process. Hence, we can make use of these variables without worrying about having to diagonalize the covariance matrix (which is precisely what we would have to do had we not used the V-Lambda filter). However, before using the eigenfactors, in order to distinguish between the measurement subsystems according to the information contributed by each one of them along the state space direction associated with the maximal estimation error (see the optimality criterion, Section II), the following mathematical preliminaries are needed.

First, for each measurement subsystem, designated by the couple (H_k^j, R_k^j), the *measurement information contribution* matrix at time t_k is defined as

$$\mathcal{J}_k^j = H_k^{j^T} R_k^{j^{-1}} H_k. \tag{15}$$

This definition is based on the definition of the information matrix for stochastic systems [13] and can be reasoned as follows. Assume, for the sake of clarity, that the system dealt with is a static system, i.e., the state of the system is a constant (in time) random vector. Then, if a sequence of N measurements are taken (using a measurement system characterized by the geometry matrix H_k and covariance R_k), the information matrix at time t_k is given by:

$$L_N = P_0^{-1} + \sum_{k=1}^{N} H_k^T R_k^{-1} H_k$$

which presents the total information of the system at time t_N as a sum of the a-priori information (represented by P_0^{-1}), and the contributions $\{H_k^T R_k^{-1} H_k\}_{k=1}^N$ to the total information made by the measurement system at the measurement update epochs t_k, $k = 1,2,\ldots,N$.

Next, for any subspace $\mathcal{L} \subset \mathbb{R}^n$ let E_s be the orthogonal projector onto this subspace, i.e., E_s is the matrix representation of the linear transformation of any vector $\mathbf{x} \in \mathbb{R}^n$ into its orthogonal projection on $\mathcal{L}$. Also, we equip the vector space $\mathbb{R}^{n,n}$ formed by all real $n \times n$ matrices (with the usual definitions of matrix addition and multiplication) with the inner product defined by the trace operator; that is, if $A \in \mathbb{R}^{n,n}$ and $B \in \mathbb{R}^{n,n}$, then their inner product is defined as [16]:

$$\langle A,B\rangle := \mathrm{tr}(B^T A). \tag{16}$$

With all this on hand, we now return to the optimality criterion defined in Section II. In order to allow for eigenvalue multiplicities, we assume that the n eigenvalues of P_k (the covariance matrix at t_k, the sensor selection epoch) take q different values, denoted by $\{\lambda_1,\lambda_2,\ldots,\lambda_q\}$. Let the multiplicity of λ_i be μ_i. Since P_k is a symmetric matrix, the geometric multiplicity of each eigenvalue is equal to its algebraic multiplicity, that is, corresponding to the eigenvalue λ_i (with algebraic multiplicity μ_i) there are μ_i linearly independent eigenvectors of P_k. These eigenvectors form a basis for the null space of $[P_k - \lambda_i I]$, denoted by $\mathcal{N}(P_k - \lambda_i I)$, which is the eigenspace of P_k associated with λ_i. Since we are interested in maximizing the input of information along the state space direction corresponding to the maximal estimation error, we first observe that this requirement can be interpreted as maximizing the measurement information contribution along the subspace spanned by the eigenvectors corresponding to the maximal eigenvalue. Denoting this eigenvalue by $\lambda_{\max}$, the corresponding subspace is $\mathcal{N}(P_k - \lambda_{\max} I)$. Let $E_{\max}$ be the orthogonal projector onto this subspace, and let $\mathcal{J}_k^j$ (15) be the information contribution of the jth measurement subsystem at t_k; then, using the preceding definitions, a scalar measure of the information contribution (by the jth measurement subsystem) along $\mathcal{N}(P_k - \lambda_{\max} I)$ is defined as

$$\rho_k^j := \langle E_{\max}, \mathcal{J}_k^j\rangle = \mathrm{tr}(\mathcal{J}_k^j E_{\max}). \tag{17}$$

For any subspace $\mathcal{L} \subset \mathbb{R}^n$ for which an orthonormal basis is available, the projector E_L onto $\mathcal{L}$ may be computed by [4]

$$E_L = \sum_{j=1}^{\sigma} \mathbf{x}_j \mathbf{x}_j^T$$

where $\{\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_\sigma\}$ is an orthonormal set of basis vectors for $\mathcal{L}$, i.e., $\mathbf{x}_i^T \mathbf{x}_j = \delta_{ij}$ where δ_{ij} is Kronecker's delta function. In particular, denoting the $\mu_{\max}$ eigenvectors corresponding to $\lambda_{\max}$ by $\{\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_{\mu_{\max}}\}$, this linearly independent set of eigenvectors can be chosen to be an orthonormal set, and, hence, may be used to compute the projector $E_{\max}$:

$$E_{\max} = \sum_{i=1}^{\mu_{\max}} \mathbf{v}_i \mathbf{v}_i^T. \tag{18}$$

Using (18) in (17) we obtain the following expression for ρ_k^j:

$$\rho_k^j = \mathrm{tr}\left[\left(\sum_{i=1}^{\mu_{\max}} \mathbf{v}_i \mathbf{v}_i^T\right) \mathcal{J}_k^j\right]. \tag{19}$$

Since the filtering algorithm used is the V-Lambda algorithm, the eigenvectors and the eigenvalues of the covariance matrix are available at all times; hence, the computation of ρ_k^j for each measurement subsystem j amounts to only using these variables in (19). Since (19) constitutes a quantitative measure of the information contributed by each measurement system along the subspace of maximal estimation error, this is what we need in order to select the optimal measurement subsystem at t_k (i.e., that system which maximizes the information contribution along the subspace associated with the maximal estimation error). That is, the optimal subsystem (denoted by its index $\hat{\jmath}$) is

$$\hat{\jmath}(t_k) = \arg\max_j \rho_k^j.$$

Remarks

1) Since the sensor selection strategy is based on identifying, first, the largest eigenvalue, the implied assumption in the foregoing analysis is that all elements of the covariance matrix have the same physical dimension (otherwise, the size of the eigenvalues might be meaningless). This can be achieved, for example, by appropriately normalizing the state vector components, i.e., by using nondimensional quantities. Note, that this common procedure is desirable also in other respects, since it can be used to yield smaller numerical range for the resulting variables.

2) Note that the measurement strategy determination consists of computing, at each sensor selection epoch t_k, the set of scalar measures $\{\rho_k^j\}_{j=1}^M$, and conducting a search for the maximum in this set. Since the use of the V-Lambda filtering algorithm facilitates the computation of the measures ρ_k^j by making the covariance eigenfactors readily available, the resulting procedure should be simple enough to be implemented in real-time in most applications.

3) The presented algorithm assumes that only one, out of the M available measurement subsystems, can be used at each measurement update. In a straightforward manner, the algorithm can be modified to treat the case where M' out of the M available measurement subsystems can be used at each measurement update (where $M > M' > 1$).

4) It is interesting to examine the role of ρ_k^j in the context of the measurement update equation. To this

end, we write the measurement update in information mode [2]:

$$P_{k/k}^{-1} = P_{k/k-1}^{-1} + H_k^{j^T} R_k^{j^{-1}} H_k \tag{20}$$

(assuming that the jth measurement subsystem is active at t_k). Then, using the spectral decomposition of the covariance [9]

$$P_{k/k-1} = \sum_{i=1}^{l} \lambda_i E_i$$

(in which E_i is the orthogonal projector onto the subspace $\mathcal{N}(P_{k/k-1} - \lambda_i I)$), (20) becomes

$$P_{k/k}^{-1} = \sum_{i=1}^{l} \lambda_i^{-1} E_i + H_k^{j^T} R_k^{j^{-1}} H_k. \tag{21}$$

Postmultiplying (21) by $E_{\max}$, the projector corresponding to $\lambda_{\max}$, and using the orthogonality property of the projectors [9], i.e.,

$$E_i E_j = 0 \qquad \text{for} \quad i \neq j$$

yields

$$P_{k/k}^{-1} E_{\max} = \lambda_{\max}^{-1} E_{\max} + \mathcal{J}_k^j E_{\max} \tag{22}$$

where the definition (15) was used. Taking the trace of both sides of (22) and using the linearity property of the trace operator yields

$$\mathrm{tr}(P_{k/k}^{-1} E_{\max}) = \langle E_{\max}, P_{k/k}^{-1} \rangle = \mu_{\max} \lambda_{\max}^{-1} + \rho_k^j$$

(where we have also used the fact that if E_s is the orthogonal projector onto the subspace $\mathcal{L} \subset \mathbb{R}^n$, then $\mathrm{tr}[E_s] = \sigma$, where σ is the dimension of the subspace $\mathcal{L}$). The last equation shows clearly that the total a-posteriori information along the eigenspace corresponding to the maximal a-priori estimation error $\langle E_{\max}, P_{k/k}^{-1} \rangle$ is comprised of the following two parts: 1) the a-priori information $\mu_{\max}\lambda_{\max}^{-1}$ along that eigenspace, and 2) the contribution ρ_k^j of the most recent measurement along that eigenspace. Therefore, maximizing ρ_k^j (by choosing the appropriate measurement subsystem) has the effect of maximizing the amount of information input to the filter along the direction of maximal a-priori error.

5) In the case of a spatially distributed measurement system, where the measurement geometry matrix is a function of both time and space, the algorithm outlined above, properly adapted to this case, may serve to optimally allocate the sensors according to the state of the estimation process. In this case a function $p_s(t_k)$ is sought, which defines the optimal location p_s of the system sensors as a function of time, i.e.,

$$\mathbf{y}_k = H[p_s(t_k), t_k]\mathbf{x}_k + \mathbf{v}_k.$$

Using standard optimization techniques this optimal location may be found by maximizing a performance index which is based on the preceding analysis:

$$J = \mathrm{tr}\left[\left(\sum_{i=1}^{\mu_{\max}} \mathbf{v}_i \mathbf{v}_i^T\right) \mathcal{J}_k(p_s)\right]$$

where the information contribution of the measurement system, $\mathcal{J}_k(p_s)$, is now a continuous function (with possible physical constraints) of the spatial coordinate p_s.

The performance of the new measurement configuration algorithm is demonstrated in the next section by way of a numerical example.

V. FILTERING EXAMPLE

In this section we present the results of a simple filtering example, in order to demonstrate the performance of the optimal measurement strategy algorithm. The dynamic system considered is described by the following time-varying discrete-time model

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + \mathbf{w}_k, \qquad \mathbf{x} \equiv (x_1, x_2, x_3)^T$$

where

$$F_k = \begin{bmatrix} 0.95 & 0.10 & 0 \\ 0 & 0.95 & 0 \\ 0.001 & 0 & 0.975 \end{bmatrix} [\sin(0.01 t_k) + 0.1]$$

$$E\{\mathbf{w}_k\} = 0$$

$$E\{\mathbf{w}_j \mathbf{w}_k^T\} = \begin{bmatrix} 0.01 & 0.0001 & 0.0001 \\ 0.0001 & 0.01 & 0 \\ 0.0001 & 0 & 0.015 \end{bmatrix}$$

$$P_0 \equiv E\{[\mathbf{x}_0 - E(\mathbf{x}_0)][\mathbf{x}_0 - E(\mathbf{x}_0)]^T\} = \mathrm{diag}[250., 250., 250.].$$

The measurement system consists of three optional discrete-time subsystems, only one of which can be used at a measurement update. These three measurement subsystems are described by the following mathematical model:

$$\mathbf{y}_k^j = H_k^j \mathbf{x}_k + \mathbf{v}_k^j, \qquad j = 1,2,3$$

$$H_k^1 = \begin{bmatrix} 1. & 0 & 0 \\ 0 & 0.001 & 0 \end{bmatrix}, \qquad H_k^2 = \begin{bmatrix} 0 & 0 & 1. \\ 0.001 & 0 & 0 \end{bmatrix}$$

$$H_k^3 = \begin{bmatrix} 0 & 1.05 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}$$

$$E\{\mathbf{v}_k^j\} = 0., \qquad E\{\mathbf{v}_k^j \mathbf{v}_k^{j^T}\} = \mathrm{diag}[0.1 \quad 0.1], \qquad j = 1,2$$

$$E\{\mathbf{v}_k^3\} = 0., \qquad E\{\mathbf{v}_k^3 \mathbf{v}_k^{3^T}\} = \mathrm{diag}[0.05 \quad 0.05]$$

$$E\{\mathbf{v}_k^j \mathbf{v}_l^{j^T}\} = 0, \qquad k \neq l.$$

The filtering problem for this system was solved with and without measurement system optimization.

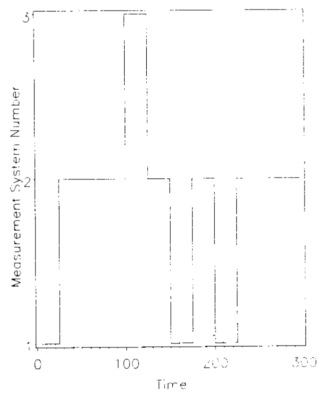


Fig. 1. Optimal measurement strategy.

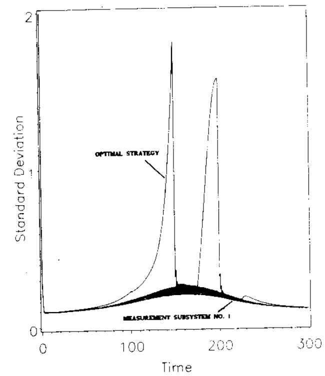


Fig. 2. Optimal strategy versus measurement system 1. Estimation error of x_1.

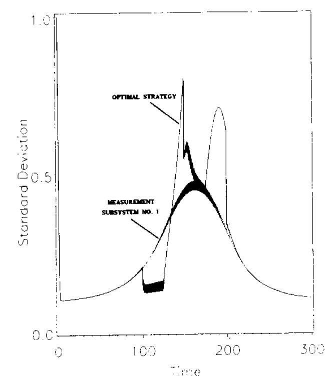


Fig. 3. Optimal strategy versus measurement system 1. Estimation error of x_2.

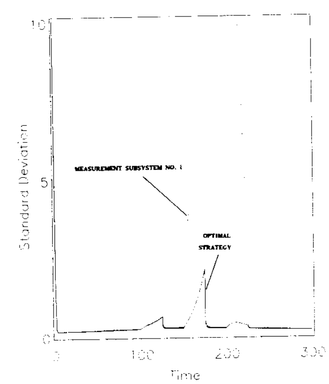


Fig. 4. Optimal strategy versus measurement system 1. Estimation error of x_3.

Four computer simulations were run. In the first one, measurement system optimization was used. In the other three runs, one measurement system was used for the complete simulation period. The filtering algorithm was the V-Lambda algorithm described in Section III. Measurements were acquired at a rate of one measurement per time step, and the simulation was carried for 300 time steps. When the sensor selection algorithm was employed, the spacing between two consecutive configuration epochs was 25 time steps.

Fig. 1 shows the optimal measurement strategy obtained by using the measurement optimization algorithm. It is seen that the optimal strategy consists of using measurement subsystem 2 for the major part of the simulation time, while subsystems 1 and 3 are used for short periods only. In Figs. 2–4 the time histories of the estimation error standard deviations of the three state variables are compared, when the optimal measurement policy is used and when subsystem 1 is used (without optimization). As can be observed from Figs. 2 and 3, the optimal measurement policy produced worse results than those obtained by using measurement subsystem 1 for the entire run. However, this can be explained easily by observing that the estimation error associated with x_3 is much larger than those associated with the other two state components; the optimal strategy, which aims at providing the maximal amount of information input along the direction of maximal estimation error, chooses to decrease the estimation error of x_3, at the price of allowing the estimation error of x_1 and x_2 to increase slightly. Similar results were obtained from a comparison of the optimal output strategy with measurement subsystems 2 and 3.

VI. CONCLUSIONS

A new measurement system configuration technique is presented for discrete-time systems with measurement system constraints. The algorithm can be implemented on-line when using the SR V-Lambda state estimation algorithm. The measurement configuration strategy is determined at each configuration epoch based on information considerations, namely: the measurement subsystem to be used is that one which provides the maximal amount of information along the state space direction associated with the maximal estimation error. The proposed procedure can be used with time-varying systems, since all that is required at the sensor selection epoch is the current state of the estimation process, and there is no explicit dependency on past or future values of the estimation variables.

Based on the V-Lambda filtering algorithm, the new technique demonstrates the advantages of using this SR formulation, which continuously provides the user with the spectral factors of the estimation error covariance.

REFERENCES

[1] Aidarous, S. E., Gevers, M. R., and Installe, M. J. (1975)
Optimal sensors' allocation strategies for a class of stochastic distributed systems.
International Journal of Control, **22** (1975), 197–213.

[2] Anderson, B. D. O., and Moore, J. B. (1979)
Optimal Filtering.
Englewood Cliffs, NJ: Prentice-Hall, 1979.

[3] Athans, M. (1972)
On the determination of optimal costly measurement strategies for linear stochastic systems.
Automatica, **8** (1972), 397–412.

[4] Ben-Israel, A., and Greville, T. N. E. (1974)
Generalized Inverses: Theory and Applications.
New York: Wiley, 1974.

[5] Bierman, G. J. (1977)
Factorization Methods for Discrete Sequential Estimation.
New York: Academic Press, 1977.

[6] Chen, W. H., and Seinfeld, J. H. (1975)
Optimal location of process measurements.
International Journal of Control, **21** (1975), 1003–1014.

[7] Deutsch, R. (1965)
Estimation Theory.
Englewood Cliffs, NJ: Prentice-Hall, 1965.

[8] Dyer, P., and McReynolds, S. (1969)
Extensions of square root filtering to include process noise.
Journal of Optimization Theory and Applications, **3** (1969), 444–459.

[9] Halmos, P. R. (1974)
Finite-Dimensional Vector Spaces.
New York: Springer-Verlag, 1974.

[10] Ham, F. M., and Brown, R. G. (1983)
Observability, eigenvalues and Kalman filtering.
IEEE Transactions on Aerospace and Electronic Systems, **AES-19** (1983), 269–273.

[11] Herring, K. D., and Melsa, J. L. (1974)
Optimum measurements for estimation.
IEEE Transactions on Automatic Control, **AC-19** (1974), 264–266.

[12] Kelley, H. J. (1962)
Method of gradients.
In G. Leitman (Ed.), *Optimization Techniques.*
New York: Academic Press, 1962.

[13] Lewis, F. L. (1986)
Optimal Estimation.
New York: Wiley, 1986.

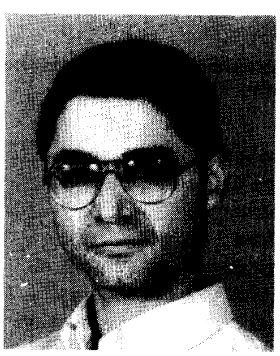
[14] Maybeck, P. S. (1979)
Stochastic Models, Estimation, and Control, Vol. I.
New York: Academic Press, 1979.

[15] Meier, L., III., Peschon, J., and Dressler, R. M. (1967)
Optimal control of measurement subsystems.
IEEE Transactions on Automatic Control, **AC-12** (1967), 528–536.

[16] Naylor, A. W., and Sell, G. R. (1982)
Linear Operator Theory in Engineering and Science.
New York: Springer-Verlag, 1982.

[17] Oshman, Y. (1989)
Gain-free information filtering using the spectral decomposition.
Journal of Guidance, Control and Dynamics, **12** (1989), 681–690.

[18] Yu, T. K., and Seinfeld, J. H. (1973)
Observability and optimal measurement location in linear distributed parameter systems.
International Journal of Control, **18** (1973), 785–799.



Yaakov Oshman was born in Israel on November 4, 1953. He received the B.Sc. (Summa cum laude) and the D.Sc. degrees, both in aeronautical engineering, from the Technion–Israel Institute of Technology, Haifa, Israel, in 1975 and 1986, respectively.

From 1975 to 1981 he was with the Israeli Air Force, where he worked in the areas of structural dynamics and flutter analysis and flight testing. In 1987 he was a Research Associate at the Department of Mechanical and Aerospace Engineering of the State of University of New York at Buffalo, where he was, in 1988, a Visiting Assistant Professor. Since 1989 he is with the faculty of Aerospace Engineering at the Technion–Israel Institute of Technology. His research interests are in advanced optimal estimation and control methods and their application in aerospace problems.